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Variability of intergranular friction and its role in DEM simulation of direct shear of an assembly of rapeseeds

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A b s t r a c t. A method for determining the coefficient of intergranular friction between pairs of metallic and organic objects of nearly spherical shape was elaborated. The method consists of simultaneous determination of both the force of friction and relative displacements of a pair of grains under normal load. This method was verified by measuring the friction force between pairs of steel plates and pairs of steel beads with a diameter of 5 mm. The optimum conditions for ensuring accuracy and repeatability of these measurements were established. Coefficients of friction of steel objects and pea, wheat and rapeseed grains at equilibrium moisture content were determined. It was found that the scatter of friction force was the highest for pea, wheat and rapeseed grains due to variability of shape, roughness of surface and irreversible deformation at the points of contact. Numerical simulations using discrete element method were performed to examine the influence of variability which the coefficient of intergranular friction had on material behaviour in a direct shear test. Two-dimensional simulations were performed assuming an assembly of 4000 circular particles having the material properties of rapeseeds with three different levels of standard deviations of μ_{pp}. Variation in the standard deviation of μ_{pp} was found to influence markedly the stress – strain characteristics while the strength of the assembly (or steady state value of stress) remained constant.

K e y w o r d s: granular mechanics, direct shear test, coefficient of intergranular friction, strength of granular material, DEM

INTRODUCTION

When two objects are in contact with each other friction forces are generated at the contact points. Several different types of friction can occur *eg* static, kinetics, rolling friction, depending on state, type of motion and contacting surfaces. It is difficult to describe friction even in the case of two flat surfaces because numerous phenomena and intervening factors are involved. Efforts to describe friction at contact points between grains face additional difficulties because of the curvature of these surfaces as well as the dependence of mechanical properties on external conditions *eg* temperature, air moisture.

Early efforts to determine the coefficient of interparticle friction, μ_{pp}, were predominately undertaken in the 1960's by soil scientists (Rowe, 1962). The goal of many of these studies was to relate the measured value to the theoretical shearing behaviour of a deposit of particulate material. Many of these tests were conducted to shed light on the numerous phenomena driven by friction between particles in contact. Recently, with the incorporation of numerical simulations as a means of deeper insight into mechanics of granular media have sparked new interest of researchers in understanding the phenomena at contact surfaces and contact model. Recent models have utilized the discrete element method (DEM) to model particulate behaviour. In DEM simulations, the trajectory and rotation of each particle within a system is obtained using a numerical time integration scheme. The contact forces at each contact point are evaluated during each step and are resolved into normal and tangential components. Newton second law of motion is then used to determine the motion of each particle from any unbalanced force. Coefficients of friction between: particle – wall μ_w and particle – particle μ_{pp} together with: density, Poisson ratio, modulus of elasticity and coefficient of restitution are crucial input parameters in DEM simulations.

Walton (1994) conducted numerical simulations to investigate the relationship between the coefficient of intergranular friction, μ_{pp}, and dynamic angle of repose, φ_r. The behaviour of material in a tumbler rotating about its horizontal axis was modelled and the coefficient of intergranualar

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friction, $\mu_{pp,}$ were found to vary over a range of from 0.02 to 1.0. The dynamic angle of repose φ_r was found dependant on the coefficients of intergranular friction, particularly for μ_{pp} lower than 0.05. At such low values of friction rotations did not play a significant role in the deformation of material within a layer and the angle of repose increased proportionally with an increase in μ_{pp}. With further increases in μ_{pp} only slight increases in φ_r were observed caused by the rotation of particles. It was found that samples comprised of nonspherical particles behaved much differently than samples comprised of spherical particles, irrespective of their μ_{pp} value. Walton concluded that angles of repose φ_r greater than 31° occurred only in assemblies comprised of nonspherical particles. Ulrich *et al*. (2007) conducted a study on the influence of the coefficient of interparticle friction on the segregation behaviour of particulate systems composed of glass and brass spheres. It was determined that variations in the coefficient of friction have a dramatic effects on the segregation of particles within granular media. Landry *et al.* (2003) performed large-scale DEM simulations on three dimensional systems of granular assemblies and determined that particle-wall friction had a significant effect on the stress state in static granular packs in cylindrical containers.

Numerous investigations have been conducted to determine factors which affect the coefficient of particle-particle friction and particle-surface friction. Lorenz *et al.* (1997) calculated dynamic interparticle friction coefficients for polystyrene, stainless steel, acrylic and glass beads, by performing experiments involving binary collision. O'Sullivan *et al.* (2002) determined surface friction between steel rods using the Thomas test (Thomas, 1997). An average friction angle of 18° and standard deviation of 2.15° were measured. The authors determined that the Thomas test could not be applied using borosilicate rods because of differences in friction in the tangential direction and longitudinal directions of the rods. For borosilicate rods a sliding block test was applied providing the average friction angle of 12.1° and standard deviation of 3°.

The majority of research on inter-particle friction has been performed using spherical or nearly spherical particles and many of these proposed methods might not be applicable to irregularly shaped particles, like agricultural grains and seeds. Moya *et al.* (2002) measured the angle of internal friction for agricultural grains using the direct shear test. Chung *et al.* (2004) applied a tilting table test to determine the static particle-surface friction coefficients μ for six different types of irregularly shaped corn grains placed on steel and aluminum plates. Higher friction coefficients were observed when using steel surfaces. The variation in the friction coefficient between each surface was less than 10% and the variation in μ between each corn type was also less than 10%. Wiącek (2008) performed a series of sliding tests using a steel plate with rapeseed, peas, beans and wheat grains. The largest friction coefficient was measured using rapeseed while the smallest was measured using bean grains. The overall variation in the friction coefficient for each grain was less than than 10%. Yurtlu *et al.* (2010) determined the coefficients of static and dynamic friction for the bay laurel seeds. Experiments were conducted varying both the moisture content of the seeds and the test surfaces. Test surfaces of galvanized steel, PVC, aluminum and chromium sheet were used. For moisture contents between 6.1 and 20.0% the coefficient of friction increased for all test surfaces. For moisture contents between 20.0 and 36.8% a decrease in both the coefficients of static and dynamic friction coefficients were observed. The largest values friction coefficients were observed for galvanized steel. A study by Izli *et al.* (2009), conducted on the influence of moisture content of rapeseed on the static particle-surface friction coefficient, showed an increase in friction coefficient with moisture content increased. The tests were performed using friction plates made from six different types of materials. At all moisture contents, rubber had the highest friction coefficient, followed by plywood, galvanized iron, glass, aluminum, and finally stainless steel. Horabik (2001) determined that both grain on grain friction and grain on wall friction was significantly influenced by the wall construction (type and roughness of the surface), normal load, sliding direction, sliding velocity, moisture content and particle orientation.

The significant increase in computing power in recent years has allowed for more efficient application of numerical methods in all engineering fields. This has also resulted in an increase in the scale of application of computational techniques in investigations of mechanical behaviour of granular materials and process design. The discrete element method (DEM), which is regarded by many modelers as one of the most promising modeling tools require μ_{pp} as one of a few input parameters.

The objectives of this project was to:
- elaborate and verify the methods of determination of coefficient of interparticle friction,
- estimate the degree of variability of μ_{pp} for three different types of agricultural grains, and
- examine the influence of the variation in μ_{pp} between seeds on the shear behaviour of an assembly of rapeseeds.

MATERIALS AND METHODS

The apparatus used to measure the coefficient of interparticle friction between two sliding particles is shown in Fig. 1 (Łukaszuk *et al.*, 2009). The function of the apparatus was to measure the horizontal force T and vertical displacement ΔH that occurs when one particle traverses over another. The apparatus allowed for rotation of a counterbalanced lever arm in both the vertical and horizontal directions. In this apparatus the lower particle of grain is static and is fixed to a lower plate which has both a controlled height and location in space. A second particle was cemented to an upper plate which was mounted on a at a distance of 30 mm from its rotary axis. The lever arm allows for movement of the upper particle in both the horizontal and vertical

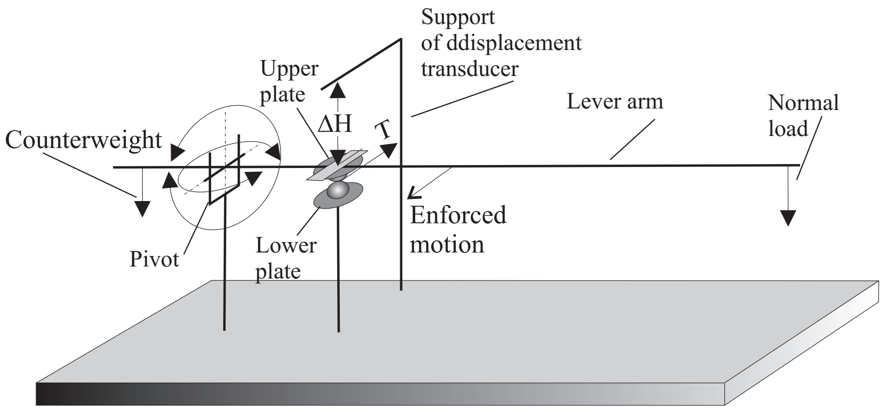

Fig. 1. Schematic of apparatus for measurement of coefficient of interparticle friction.

direction. During testing the upper particle traverses over the lower particle. The normal pressure between these particles were allowed to vary by varying the location of a weight on a lever arm. A counterweight was located on a shorter arm to balance the uneven weight of the pair of lever arms. The motion of the lever in horizontal plane was enforced by the string connected to a drive of adjustable velocity through a load transducer that measured horizontal force. The measurements of both horizontal force and vertical displacement of upper plate were measured simultaneously using an inductive transducer. The measurement method was verified through measuring friction coefficients between pairs of flat brass or steel square plates of 10x10 mm sides, dry or lubricated steel beads of diameter of 5 mm and pairs of pea, wheat and rapeseed seeds. Rapeseeds were tested at normal load of 0.59 N while the other samples were analyzed at normal load of 0.98 N.

DEM simulations were obtained using the code adopted from Wassgren (1997) for 2D systems. Circular elements with diameters uniformly distributed between 1.8 and 2.2 mm and density of 1 000 kg m^{-3} were modeled within the 2D system. Interaction of the particles in the normal direction was simulated using a linear viscoelastic model. A spring constant k_n of 2 10^4 N m^{-1} was used to represent the relatively soft particles like rapeseed. The spring constant is based on values measured by Stasiak *et al*. (2007) on the modulus of elasticity of uncompacted bulk rapeseed over a range from 10 to 20 MPa. The dumping constant β_n of 0.14 N s m^{-1} was used to reflect a restitution coefficient of 0.76, which is a typical value for these materials. To model interaction in the tangential direction the linear viscoelastic model was expanded to include a frictional element. A friction coefficient of 0.3 was assumed for the entire shear test for the deterministic simulations or was treated as a random variable following the Gaussian distribution with the mean value μ of 0.3 and standard deviation σ of 0.03 or 0.06. In the case of the probabilistic contact model values of the friction coefficient were generated individually for each new contact. The friction coefficient was assumed constant throughout the contact duration. The simulations were performed using a time step of Δt of 2.8 10^{-7} s which allowed numerical stability in the calculations.

A direct shear test was simulated in 2D assuming a 200 mm long shear box having a height to diameter ratio *H/D* of 0.43. The 4 000 particles were generated randomly in the box of 1000 mm high and 200 mm long. Particles free to rotate and move were dropped down onto the bottom of the shear box under gravitational force. After resting of particles the top cover of the shear box with the dead weight of 16 N was placed on the free surface of the assembly and the system was left to rest. The top cover was allowed to move in the vertical direction without rotation. The 2D porosity of the system was equal to 0.167. The shear process was modeled by moving the bottom part of the shear box in the lateral direction at a rate of 1.4 10^{-3} m s^{-1} under a constant vertical load, $\check{N}$, exerted by the weight of the top cover. The tangential force in the lateral direction, , was determined as the sum of lateral forces (normal to vertical walls and tangent to the apparatus floor or top cover) exerted on the upper or lower part of the shear box.

RESULTS AND DISCUSSION

A typical relationship between the friction force, *T*, and displacement, Δl, measured during testing using two flat steel plates under a normal load of 0.98 N is shown in Fig. 2. Fluctuations typical of those termed as stick-slip were observed during testing. During the stick-slip process the friction force builds up during the stick phase and then decreases significantly during the slip phase. At an average friction force of 0.25 N amplitudes of vibration of greater than 0.02 N, were observed. Except of those weak and relatively regular vibrations higher and more irregular disturbances in $T(\Delta l)$ experimental curves were noted that reached few percent with respect to mean value. This was probably caused by unevenness of the sliding surfaces. It was

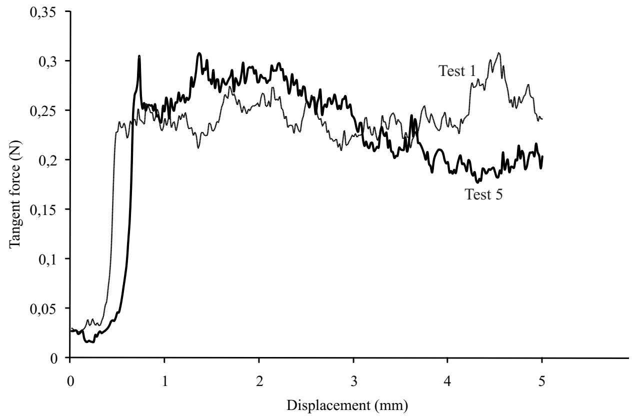

Fig. 2. Friction force, *T*, versus horizontal displacement *l* for first and fifth repetition of test measuring coefficient of friction between two steel plates under normal load of 0.98 N.

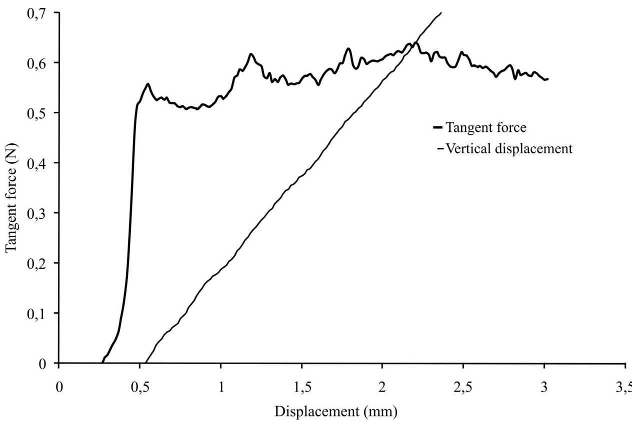

Fig. 3. Relationship between force of friction, *T*, and horizontal displacement, *l*, obtained from test measuring coefficient of friction between steel ball and steel plate under an inclination angle of 20º, under normal load of 0.98 N.

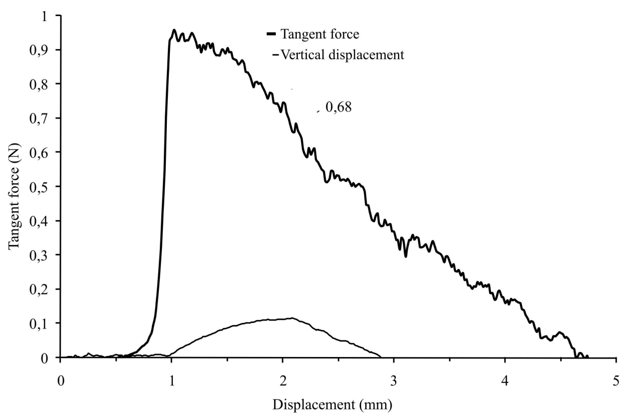

Fig. 4. Force of friction, *T*, versus vertical displacement, *ΔH*, for the test measuring coefficient of friction between two steel balls of diameter of 5 mm.

very difficult to replicate this effect from one test to another because of the different frictional conditions that occurred in subsequent trials with different friction conditions even with the same material. Friction coefficients obtained for steel and brass at a relative displacement between 2 and 3 mm were 0.16±0.014 and 0.25±0.000, respectively.

The tangential force measured using this device is comprised of frictional and shape components. Tests involving steel beads sliding against a steel plate inclined at an angle of 20º were performed to examine the sensitivity of the equip ment to surface inclination. Typical results of the tangential force measured as a function of horizontal displacement ($T(\Delta l)$) under a normal load of 0.98 N is shown in Fig. 3. A sharp ramp in horizontal force at the start of movement was observed. This rapid increase was followed by a region in which the force stabilized at an average horizontal force value of 0.6 N. In this region much smaller fluctuations in the force were observed. This flucations were believed caused by the non-perfect flatness of steel plate. The coefficient of friction μ and angle of friction atan (μ) were determined to be 0.63±0.03 and 32º, respectively. Summation of friction angle of steel calculated from earlier determined μ_{pp} of 0.25 (14º) to inclination angle of plate (20º) gives 34º which is fairly close to experimental value of atan(μ). These results show that the apparatus measures the tangential motion resistance accurately as a sum of a frictional force and a force resulting from inclination of the sliding path.

A typical relationship between the frictional force *T* and vertical displacement *ΔH* obtained for two steel beads sliding under normal load of 0.98 N is shown in Fig. 3. The applied normal force corresponds to a stress of 40 kPa which is the same stress exerted on a flat bottom bin with 8 m high of wheat bedding. The $T(\Delta H)$ relationship illustrates that the horizontal force resisting motion is a sum of the force necessary to overcome the inclination of surface of upper bead with respect to lower one and a frictional resistance force. The shape component of the motion resistance is equal to zero when the lowest point of the upper bead traverses over the top of the lower one. Then direction of displacement *ΔH* is changing and shape component of force of motion resistance acts opposite to frictional force. Theoretically, the maximum value of *ΔH* is reached when the tangential force is equal to the frictional force. In reality the path of relative motion of one bead with respect to another may not run along the maximum meridian and then the resultant force is not equal to the force of friction. This effect was included in the analysis and its influence was limited through precise positioning of beads before measurement. Another factor which effects these measurement is the elasticity of the construction members which comprise the apparatus. The elasticity of these members can accumulate energy during these tests and then release it. That effect can be seen in Fig. 4 as a step increase in frictional force at the beginning of this test. The frictional force is also affected by the differences in static and kinetic friction. Static friction is usually higher than kinetic friction. Kinetic friction can be observed when motion commences and it normally results in a pressure peak. The higher force is required to initiate motion in comparison to force required to maintain motion. The magnitude of the *T*(*ΔH*) force decreased monotonically during these experiments accompanied by fluctuations in the force *T*. This is typical sliding with friction.

Particle on particle coefficients of friction (μ_{pp}) between beads were calculated using the frictional force at the point where the vertical displacement, *ΔH*, was maximum. However, for some tests a sharp increase in the force, *T*, occurred at the same time as that of the maximum displacement. For those cases the frictional force measured before a violent decrease occurred was used in the calculations. For these tests the friction coefficients varied over a range from 0.69 to 0.5, with a mean and standard deviation value of 0.52±0.04 obtained from the six repetitions.

Additional tests were conducted using the same pair of beads to observe the effect that the length of sliding path had on the friction coefficient. The particle on particle friction coefficient (μ_{pp}) was observed to decrease in subsequent tests as a consequence of the wear-in of contacting surfaces. Presence of normal and tangential loads in a contact area changes the state of surfaces and, as a result the frictional force. After a significant number of tests the friction forces stabilized as *eg* in the case of vertical wall loads measured in smooth wall bin (Molenda *et al.*, 1996). Other factors which have been shown to affect the frictional conditions between sliding surfaces are temperature, moisture content or chemical activity of ambient of experiments.

Frictional force were also measured for a pair of steel beads lubricated with mineral oil. A typical graph is shown in Fig. 5 of the horizontal force, *T*, and horizontal displacement, *Δl*. The presence of oil on the contact surfaces resulted in a twofold decrease in the maximum motion resistance force. For lubricated beads this force varied from 0.2 to 0.3 N while for dry beads this force varied over a range from 0.5 to 0.7 N. The graphs describing these relationships *T*(*Δl*) also varied for these two conditions. Less frictional vibrations were observed in the case of lubricated steel beads.

Coefficients of friction for the lubricated steel beads were found to vary over a range from 0.21 to 0.16 with a mean and standard deviation value of 0.19±0.01. These results indicate the possibility that exists of strong variation in the coefficient of friction even on theoretically identical surfaces. While the surface was lubricated, even the lubricated surface can have various amounts of lubrication at various locations which can results in much different surface properties. Therefore, no reliable method exists for estimating the coefficient of friction based on surface characteristics.

The *T(Δl)* characteristics measured for pairs of wheat, pea and rapeseed seeds revealed features similar to those obtained for steel beads, but considerably higher fluctuations were observed. Rapeseeds acted similar to the steel beads and the variation between pairs of these type seeds was the smallest. Figure 6 shows a typical graph of *T(Δl)* for pea grains, which had the greatest variation of all other seeds. The surface roughness of the contacting pea seeds were believed to cause large deviations shown in Fig. 6. The ramping up in the horizontal force were thought to occur when contact occurred between two asperities of high inclination, while the ramping down in this same force took place after surpassing the highest contact point.

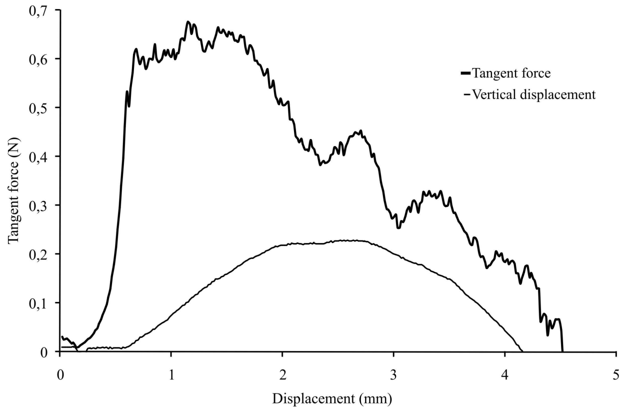

Fig. 5. Horizontal force, *T*, *vs.* vertical displacement, *ΔH*, and horizontal displacement, *Δl*, for two lubricated steel beads of diameter of 5 mm.

Fig. 6. Relationship between force of friction, *T*, and horizontal displacement, *Δl,* for pair of pea grains.

Coefficients of friction for those materials tested in this paper are shown in Table 1. The variation in friction coefficients for plant materials was higher than that of metallic objects. This is believed caused by differences in surface state and unrepeatable sliding path which occurred during measurements. For different pairs of grains, even of the same species, or even at various locations the surfaces of the same seeds can vary considerably.

High repeatability of results for seed materials with little variation can only be ensured through thorough selection of seeds and unification of contact conditions. This

T a b l e 1. Coefficients of friction and standard deviations of metal objects, pea, wheat and rapeseed grains

Material	Coefficient of friction (standard deviation)
Brass plate	0.16 (0.01)
Steel plate	0.25 (0.01)
Rough steel ball	0.52 (0.04)
Smooth steel ball	0.19 (0.01)
Pea	0.29 (0.09)
Wheat	0.15 (0.05)
Rapeseed	0.23 (0.03)

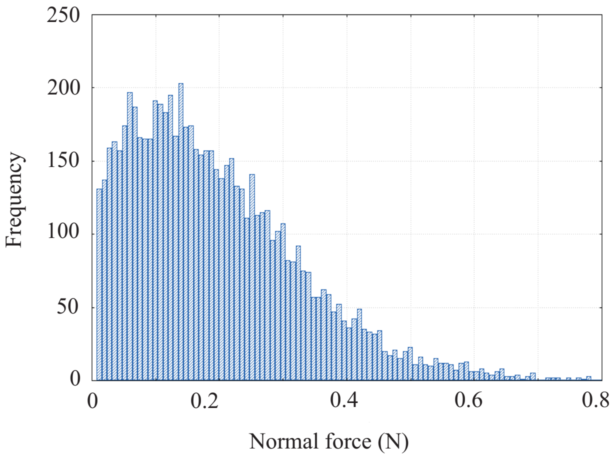

Fig. 7. Distribution of normal force at contact points for shear strain $\Delta L/L=0.0$ and random distribution of coefficient of inter-particle friction μ_{pp} = 0.3±0.06.

would also require a significantly higher number of repetitions than those conducted in this experiment, which would be unpractical. A possible solutions to this problem would be could be estimating values of of μ_{pp} from the angle of internal friction or angle of repose.

Numerical simulations using discrete element method were performed to examine the influence of variability of coefficient of intergranular friction on material behaviour in direct shear test. The coefficients of intergranular friction measured in the initial part of this study was used in the simulation. Two-dimensional simulations were performed on an assembly of 4 000 circular particles having material properties of rapeseeds with three levels of standard deviations of μ_{pp}. Variations in standard deviation of μ_{pp} was found to influence markedly the stress – strain characteristics while the strength of the assembly (or steady state value of stress) remained constant.

The distribution of the normal force prior to the start of horizontal motion is shown in Fig. 7. This force represents the inter-particle contact force immediately after filling of the apparatus for the case of a random distribution of particles. The value of force with the greatest frequency was 0.16 N, which had a frequency of approximately 200. This value of force is equal to the mean normal force, which represents the total normal load acting on the cover of the apparatus dived by 100, which is the number of particles in the highest layer of the assembly.

The distribution of normal and tangential forces for materials in bulk at a shear strain of $\Delta L/L$ of 0.1 is illustrated in Fig. 8. The thicker black lines correspond to higher values of force. This figure shows the characteristic non-homogeneous distribution of normal forces with a concentration of these forces along the line connecting the lower left and upper right corners of the box. Due to the nature of friction an infinite number of contact forces are possible between 0 and the force μN, at the sliding points. In other words, at the contact points between two granules the amount of friction which is mobilized is unknown until sliding commences. The maximum value of these contact forces is not constant and depends on the location of the contact area at the surface of the granule, on the load history *eg* wear-in, direction of sliding *etc*. or chemical interactions between the contacting bodies and/or the environment. In the case of biologically based materials humidity is an essential factor because material properties change with wetting or drying of these products. As shown in Figs 8b, c and d friction at the contact points at fully developed yielding ($\Delta L/L$ = 0.1) was mobilized to various degrees. Contacts with nearly fully mobilized friction ($T > 0.999\,\mu$ N, Fig. 8d) were located mainly in the central part of the shear box, where the normal forces were the highest (see Fig. 8a).

Simulations of the tangential to normal force (/Ň) as a function of shear strain ($\Delta L/L$) are shown in Fig. 9. A coefficient of interparticle friction of 0.3 was assumed with three different values of standard deviation of 0.00, 0.03 and 0.06. The three curves each increased with an increase in displacement and reached a horizontal asymptote of a /Ň value approximately equal to 0.35. However, each of these curves is distinctly different in the initial portions of the shear path up to a strain value of approximately 0.02. The curve for deterministic μ_{pp} = 0.3±0.00 reached asymptotic value very fast at a strain of approximately 0.003, while in the case of the highest variability μ_{pp} = 0.3±0.06 the strain of approximately 0.03 was required to reach asymptotic value. Such an effect is well known in mechanical testing of granular materials with various degree of consolidation. Well consolidated, dense samples require a shorter shear path to reach steady state flow than loose beddings. Eber (2004) in analysis of results of uniaxial compression of 2D assemblies of cylinders described these states as high levels of organization (HLO) as opposed to low levels of organization (LLO). In these experiments a well distributed stress pattern (termed by Eber (2004) as completed compaction) was achieved after 20% of the unidirectional deformation had occurred.

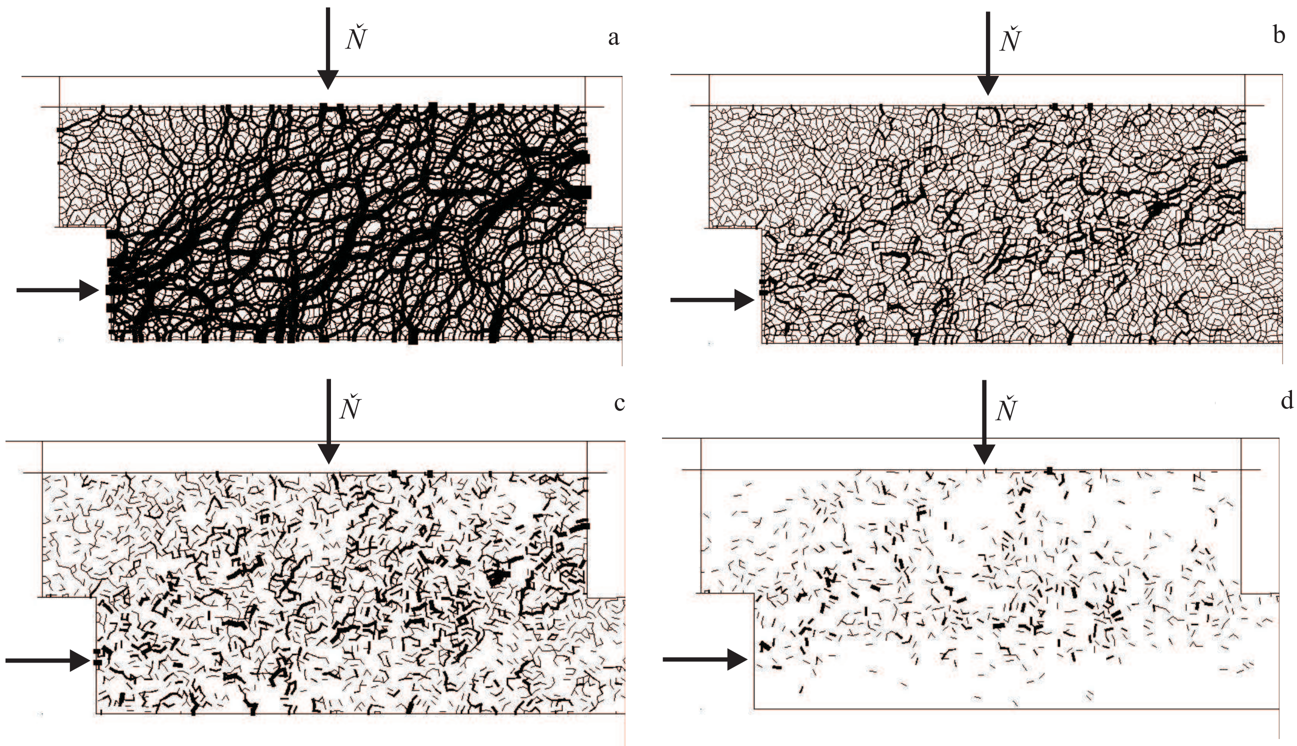

Fig. 8. Distribution of normal, Ň, and tangent, , forces at contact points for shear strain $\Delta L/L=0.1$ and coefficient of interparticle friction $\mu_{pp} = 0.3\pm0.00$: a – normal forces, b – tangent forces > 0.001 μ Ň, c – > 0.6 μ Ň, d – > 0.999 μ Ň.

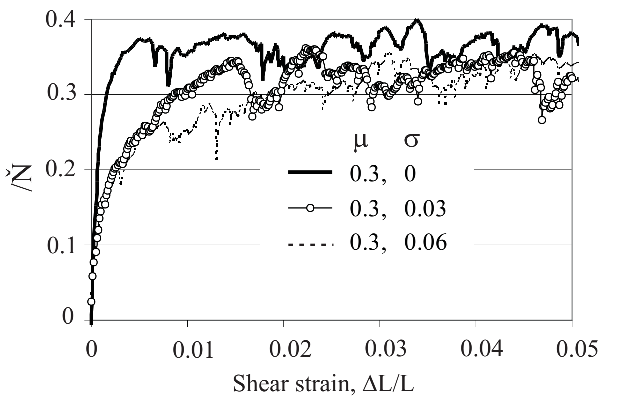

Fig. 9. Relationship of /Ň versus shear strain $\Delta L/L$ for constant $\mu_{pp} = 0.3$ and random values of coefficient of interparticle friction ($\mu_{pp} = 0.3 \pm 0.03$ or $\mu_{pp} = 0.3 \pm 0.06$).

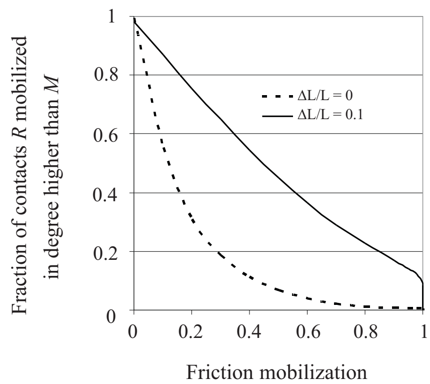

Fig. 10. Fraction of contacts R with shear friction mobilized in degree higher than M versus degree of friction mobilization M for $\Delta L/L = 0$ and $\Delta L/L = 0.1$.

For much shorter motion random compaction was not completed. The high surface roughness was also determined to be an important factor impeding compaction. An assembly of smooth cylinders of regular shape was determined to be compacted after a deformation of 5%, while other types of cylinders required a longer shear path.

Degree of mobilization of friction forces in the sample understood as a distribution of quotients M of actual tangent forces, T, in contact network to maximum possible tangent contact forces, μN, depends on the type of consolidation stress. Figure 10 shows a fraction R of contacts with friction mobilized in degree higher than M to total number of contacts compared to degree of mobilization of friction force M for two assemblies: consolidated under only vertical load ($\Delta L/L = 0$) and at fully developed yielding ($\Delta L/L = 0.1$). In the case of sample consolidated under only vertical load the fraction R decreases hyperbolically with an increase in M, while with the shear stress acting, in the state of fully developed yielding R decreases nearly linearly, and then for M>0.99 very high and narrow ramp of fraction of fully mobilized contacts is observed. This ramp in fraction of contacts mobilized in the highest degree consist small fraction of the total number of contacts (R≈0.09). Average value of the fraction M was found of 0.18 for vertically consolidated sample while it was found of 0.5 for sample at fully developed yielding. This result suggests that average degree of mobilization of friction can serve to build a general index of proximity of a granular system to yielding.

CONCLUSIONS

1. Measurements of the coefficient of friction between flat metal plates gave acceptable repeatability. Based on six test repetitions the coefficients of friction of brass and steel were determined to be 0.16±0.01 and 0.25±0.01, respectively. Measurements performed using steel beads and an inclined steel plate showed that the apparatus adequately measured horizontal force which was comprised of both frictional and shape components. More variation was observed in the coefficient of friction for tests conducted using dry steel beads, with the coefficients of friction varying over a range of from 0.44 to 0.56 with a mean and standard deviation value of 0.52±0.04. This higher amount of variability was thought to be caused by variations within the contact area caused by both non-homogeneity of frictional properties of the surfaces and the fact that the exact sliding path could not be repeated from one repetition to another. In the case of lubricated beads a smaller amount of variability was noted in the friction coefficient with a mean and standard deviation value of 0.19±0.01.

2. Measurements of the coefficient of friction of seeds *ie* pea, wheat and rapeseed produced larger amounts of variability than those of metal surfaces. In the case of seeds, this additional amount of variability was thought to be caused by the variation in shape of the seed, seed surface asperities, as well as the potential for plastic deformation of the seeds at the contact points. Even small irregularities in the surfaces of the contacting objects resulted in an immediate cease of sliding and ramp in horizontal force that might not happen in next measurement at different sliding path. Mean and standard deviations values of μ_{pp} for peas, wheat and rapeseed grains were determined to be 0.29±0.09, 0.15±0.05 and 0.23±0.03, respectively.

3. To reflect the natural variability in the frictional properties of seeds a random distribution was assumed in modeling the coefficient of friction during simulations of direct shear test of rapeseeds. A coefficient of intergranular friction of 0.3 was adopted with three levels of standard deviation of: 0, 0.03 or 0.06. The degree of variation of μ_{pp} was determined not to influence the final value of shear strength at steady state flow. However, the level of standard deviation of μ_{pp} was found to markedly influence the shear path at the initiation of motion. The deterministic assembly reached an asymptotic value at a strain of approximately 0.003 for a standard deviation of zero, while the assembly with the largest standard deviation of μ_{pp} of 0.06 required the longest strain of approximately 0.03.

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