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CRACK WIDTH, CROSS-SECTION AREA, AND VOLUME IN SWELLING CLAY SOILS*

V.Y. Chertkov, I. Ravina

Faculty of Agricultural Engineering, Technion, Haifa 32000, Israel

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A b s t r a c t. A model is presented for prediction of crack network geometry in clay soils. The starting point is the similarity between a crack network in rocks and that in soils, and, hence, principles of multiple crack formation in rocks can be used. The model estimates the mean crack width, crack cross-section area, and crack volume. Special verification of the model was done with published data on these characteristics in the micro-shrinkage depth range. Agreement between the data and the model predictions for these depths is satisfactory.

K e y w o r d s: geometry, swelling soils, shrinkage, cracks

INTRODUCTION

Drying clay soils can be divided into two interacting subsystems: the soil matrix and the shrinkage crack network. Modelling the geometry of a crack network in a swelling clay soil is an important step in describing the hydraulic properties of such soils, preferential flow, and solute transfer. We may distinguish between the upper soil layer of macro-shrinkage with cracks of widths > one mm and the lower soil layer of micro-shrinkage where crack widths < one mm. The major objectives of this work are to propose a model of the geometrical characteristics of cracks in drying clay soils and to verify the model predictions by using data available in literature on crack widths and specific volume in the depth range of micro-shrinkage.

A shrinkage crack is characterized by its width and two dimensions of its surface, appreciably larger than the width (for vertical cracks these two dimensions are the crack depth and the length of its trace at the soil surface). In this work "crack dimension" stands for the maximum dimension of a crack (we assume that for a vertical crack it is its depth).

Orientation, position, and dimensions of a separate crack in a homogeneous brittle material can be predicted, in principle, if the external stress field is known [4]), and in this sense are non random. However, in case of multiple cracking in a volume and/or random distribution of strength properties of material (heterogeneity) these crack characteristics can not be predicted (before actual measurements) and are described by distributions of probability density (see, e.g., [14]) or averages on distribution. In this presentation the crack network geometry implies the distribution of cracks, their volume and cross-section area in a soil volume, and the distribution of their dimensions.

INFLUENCE OF WATER CONTENT ON CRACK NETWORK

In continuum mechanics, flow in swelling soils, unlike in soils with rigid matrix, is usually described by the material coordinate approach (see, e.g., [1,15]). The water content profile and its variations in time play a principal role

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in such a description. Shrinkage cracks, appearing in the drying process, are not treated separately, but considered as a part of the structural porosity. Our aim is to describe explicitly the shrinkage-crack network as one of two interacting subsystems of swelling soil.

Let us consider an effect of water content profile in cracking by shrinkage. Its influence on deformation and cracking of drying clay soils manifests through a volume shrinkage, $\delta V(z)$, a horizontal-surface shrinkage, $\delta(z)$, and a vertical shrinkage, $\varepsilon(z)$ of soil matrix as functions of soil depth. In case of isotropic shrinkage (considered below) these functions are determined by the water content profile and shrinkage curve of the soil matrix. Volume shrinkage, δV consists of two contributions corresponding to a soil subsidence and a volume of appearing cracks [3]. Both cracking (that is all characteristics of crack network) and subsidence at any soil depth are functionals of functions $\varepsilon(z)$ and $\delta(z)$. However, in the hypothetical case where $\delta(z)=0$ there is only subsidence, and in the opposite case where $\varepsilon(z)=0$ there is only cracking. So one may assume that the influence of horizontal-surface shrinkage $\delta(z)$ on cracking is rather stronger than that of vertical shrinkage $\varepsilon(z)$. Accounting for also that at all depths as a rule $\varepsilon<<1$ and $\delta<<1$ we can consider that geometrical characteristics of a shrinkage-crack network depend (as functionals), in the first approximation, on the horizontal-surface shrinkage only. Consequently, regarding crack network geometry in frame of this approximation we can use the usual concept of soil depth as measured relative to the soil surface (as in experimental works [9,17,18]).

MODEL OF MULTIPLE CRACK FORMATION

There is a far-reaching similarity between cracks in rocks and in swelling clay soils, if dry enough. In both systems, in spite of their differences: a) cracking is related to a multiple crack formation, typical of which is the large number of cracks in a given volume and their small dimensions compared with the characteristic dimension of the volume; b) cracking, accompanied by crack opening, is of the same physical nature and is caused by superficial cooling (igneous rocks) or drying (sedimentary rocks and swelling clay soils); c) crack systems that divide or almost divide the volume into blocks (for rocks) or peds (for soils) are developed; d) the distribution of the intervals between the intersections of the cracks with an arbitrary line is related to one type [9, 12]; e) inhomogeneity of cracking conditions results in regular spatial changes of crack and block (ped) patterns (e.g., the changes of the block dimension distribution with rock depth and the ped dimension distribution with soil depth). Peculiarities of a clay soil cracking are connected with the cyclic (seasonal) nature of the shrinkage-swelling processes. Nevertheless, the generality, mentioned above, suggests that the spatial distribution of cracks (in homogeneous conditions, i.e., locally) and the crack dimension distribution (except the crack width) can be, in both systems, described within a framework of common concepts. In earlier works Chertkov [5,6] proposed a model of multiple crack formation to describe cracking and fragmentation of rocks. Using data of Guidi *et al.* [11] he applied the model to describe statistically homogeneous cracking of soils [8]. The rock cracking model is used here as the basis for a model of crack network geometry in clay soils that takes into account the shrinkage-swelling transitions and the changes that depend on depth. First, let us summarize the major concepts and relationships of the basic model [5,6].

Multiple cracking and fragmentation concepts

It is assumed that cracking is statistically uniform and that a fragment, as well as a crack, is characterized by a single dimension (among the cracks forming the faces of a fragment there is one of a maximum dimension - the fragment is characterized by this dimension). Multiple crack formation is the concept implying the following three points: a) participation (in the process of crack accumulation) of cracks of all possible dimensions starting from microcracks; b) randomness of the crack arrangement; and c) effective independence of cracks. These are shortly discussed in the following.

The participation of cracks of all possible dimensions

The basic concepts of disk-shaped micro-cracks of any orientations in a volume of material, proposed and validated by Zhurkov *et al.* [19,20] are: a) for a brittle material the characteristic dimension of micro-cracks (their diameter), l is considered to be constant (for clay soils one can consider l to be that of a clay particle, <2 μm); b) in a small enough volume the distribution of micro-cracks is uniform, in a uniform stress field; c) in a large enough volume random clusters of micro-cracks are formed, and their uniform distribution is destroyed; d) as the mean concentration of micro-cracks in the 3-dimensional cluster, n increases to a critical value, n_* instability occurs when the ratio (K) of the linear dimension of the average volume occupied by one micro-crack ($n^{-1/3}$) to the dimension (l) of the micro-crack itself, i.e., $K=n^{-1/3}/l$, decreases to the critical value, $K_*=n_*^{-1/3}/l$, below which an avalanche-like crack coalescence occurs; e) based on data for various materials K_*=2.56-6; for rocks and soils $K_*\cong 5$; f) the value K_* is the same for the coalescence of micro-cracks of different orientations. For macro-cracks >>l, Zhurkov *et al.* [19,20] found that the critical value, K_* is independent on scale and is applicable to clusters of cracks of any dimension. Consequently, a crack of the dimension x is, on the average, considered to result from a number of sequential coalescences of increasingly larger cracks within an initial crack-like cluster of micro-cracks. Hence, the dimension x is defined by the number of sequential coalescences, i and the size of a minimal cluster of two micro-cracks, ranging from ~$2l$ to ~ $(K_*+1)l$. The final dimension x of the crack maybe written in the form [5]:

$$x=(K_*+1)^i l, \qquad i\geq 0.3 \qquad (1)$$

where i is not only the integer, but any number, beginning from $i\cong 0.3$, for which $x\cong 2l$ (minimal cluster). According to Eq. (1) cracks whose dimensions differ by less than (K_*+1) times are considered to be of a similar dimension. The interval of change in the dimension of "similar" cracks bounding a fragment, Δx corresponds to the unit interval of change in the parameter i: $\Delta x \leftrightarrow \Delta i = 1$.

The randomness of the crack arrangement

Taking into account the above and the random crack arrangment, we can introduce: a) a local concentration of "similar" cracks in the range $\Delta i = 1$, $\hat{N}_c(x)$, as the derivative of the concentration of connected cracks of all dimensions <x with respect to $i(x)$; b) the intuitively obvious condition that similar cracks of an "average" dimension x are connected, or almost connected, and form a fragment of the same dimension, if their local concentration $\hat{N}_c(x)$ is such that the dimension of the area taken by one crack does not exceed the dimension of the crack itself: $\hat{N}_c(x)^{-1/3}/x \leq 1$; and c) an average concentration, $N_c(x)$ of connected cracks (the average of the random quantity, $\hat{N}_c(x)$) and an average cracking, $I(x) \equiv N_c(x)x^3$ (the average number of connected cracks of dimension x in a volume of x^3).

Effective independence of cracks

In the case of multiple crack formation the interaction between two separate cracks that develop under the action of shrinkage stresses are assumed to be small compared with the effect of all the cracks surrounding any one of them. In this meaning, cracks are independent. This is confirmed by data of Hudson and Priest [12] and Scott *et al.* [14] and leads to a Poisson distribution of the number of cracks of a given dimension in a randomly selected volume [10].

Expression for average cracking

Let us consider a network of intersecting cracks and assume that the average spacing, d between neighboring intersections of cracks with a straight line is similar at all orientations of the line. Then based on the data of Hudson and Priest [12], Scott *et al.* [14], and the effective independence of cracks, we may use the approximation, $\exp(-x/d)/d^3$, for the average concentration, $\int_x^{\hat{x}} N(x)dx$ of all types of cracks (both connected and isolated of any orientation) of dimensions >x, where $\hat{x}$ is the maximum crack dimension; $N(x)$ is the average concentration of cracks in a unit interval around x value. Then, the average concentration of all types of cracks with dimensions between x and $x+dx$ is the differential of $\exp(-x/d)/d^3$ with respect to x:

$$N(x)dx = \left(\exp(-x/d)/d^4\right)dx\,. \qquad (2)$$

The concentration of the connected cracks of all dimensions is some fraction, c of the concentration of all cracks. This quantity, $c \leq 1$ will be referred to as connectedness. According to the definitions of $N_c(x)$, $N(x)$, and c:

$$\int_x^{\hat{x}} N_c(x)di(x) = c\int_x^{\hat{x}} N(x)dx\,. \qquad (3)$$

Differentiating Eq. (3) with respect to x and accounting for Eqs (1) and (2), one can obtain the expression of the average cracking, $I(x) \equiv N_c(x)x^3$ (in the 3-dimensional case):

$$I(x) = \ln(K_* + 1)c(x/d)^4 \exp(-x/d)\;. \qquad (4)$$

Probability of connection of cracks

The above condition of connection of cracks and the Poisson distribution lead to the probability, $f(x)$ of connection of cracks of any orientation of dimension x [5]:

$$f(x) = 1 - \exp(-I(x)). \qquad (5)$$

The maximum dimension, x_m of the fragments is found as the point of maximum of $I(x)$ [5]:

$$x_m / d = 4, \qquad (6)$$

and the fragment formation probability, f_m, as derived from Eqs (4) and (5) at $x = x_m$ [6], is:

$$f_m \cong 1 - \exp(-8.4c). \qquad (7)$$

Equations (4) - (7) describe connected cracks which are the surfaces of fragments. In a two-dimensional case (e.g., a horizontal cross-section) it follows from the general approach that the power of the argument x/d in Eq. (4) as well as the right hand side of Eq. (6) should be 3 instead of 4 [8].

Feasibility of the model was verified by data on natural rock blocks as well as their blast fragmentation [5,6]. Its applicability to cracking soils was checked in part in works [7,8].

AVERAGE CRACK SPACING AND CONNECTEDNESS AS FUNCTIONS OF DEPTH

The above model describes statistically-homogeneous cracking. In real soils crack concentration changes with depth. This means a dependence of the characteristics d (or x_m) and c (or f_m) on soil depth z. We assume that in a clay soil the crack network geometry is determined solely by the vertical water content profile. We define a "shrinkage layer" of thickness z_s as a layer with macro-cracks of a depth that can be measured by a flexible wire [9,17,18] (Fig. 1). An important parameter of the model is the maximum crack depth, z_m. Physically $z = z_m$ is the depth where connected

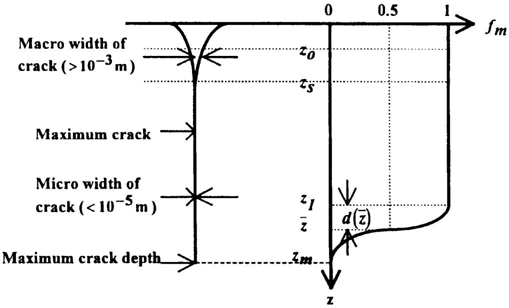

Fig. 1. Scheme of a cracked soil layer (horizontal cracks are not shown): 0 - the soil surface; z_o - the depth of the boundary of the intensive-cracking layer; z_s - the "shrinkage layer" depth; $z_1 \cong 0.75\bar{z}$; $\bar{z}$ - depth averaged on the probability $f_m(z)$; $z_m \cong 1.25\bar{z}$ - the maximum depth of cracks; $d(\bar{z})=\bar{z}/4$ - the half width of a transitional layer.

cracks and separated peds cannot appear (Fig. 1) so that:

$$f_m(z_m) = 0. \tag{8}$$

We assume that in the limiting case the boundary z_m is the water-table level. The difference $z_m - z_s$ is about the size of the capillary fringe which according to Bouma and Loveday [2] is:

$$z_m - z_s \cong \text{several meters}. \tag{9}$$

Shrinkage increases closer to the soil surface and there is an upper layer (of a few tens of cm) of intensive cracking of a depth z_o (Fig. 1) that we define by the following condition:

$$d(z_o) = z_o. \tag{10}$$

This condition provides a measurable criterion to determine z_o and means that the average spacing d between neighboring intersections of cracks with a straight line at depth z_o is equal to the latter. Thus, the depth z_o is another important parameter of the model. We assume that (Fig. 1):

$$z_o \le z_s. \tag{11}$$

This assumption will be checked below by comparing estimates of z_o with data on z_s (Table 1, case numbers 1-8). It should be noted that entered parameters z_m and z_o are functionals of variation of the horizontal surface shrinkage, $\delta(z)$ with depth. Explicit form of these functionals here is not used. Except that, the horizontal surface shrinkage enter explicitly in the definition of crack width (see below Eq. (28)).

The probability $f_m(z)$ of finding a separate ped diminishes from a maximum at the soil surface to zero at $z = z_m$ (see Eq.(8) and Fig. 1). One may introduce a depth, $z = \bar{z}$ averaged on this probability (Fig. 1) which gives the maximum depth where in the volume $\sim (x_m(\bar{z}))^3$, on the average, one may still find a separate ped or else connected cracks. This parameter will be expressed through z_m.

At the soil surface the number of cracks is very large, and one can assume that $x_m(z) \cong 0$ for z=0. Considering also the monotonic increase of $x_m(z)$ with depth, one may approximate it as:

$$x_m(z) = A(z/z_o)^{\bar{\omega}} \tag{12}$$

where A and $\bar{\omega}$ are assumed to be constants. According to Eqs (6) and (10) the maximum ped dimension at the depth $z = z_o$ is:

$$x_m(z_o) = 4z_o. \tag{13}$$

It is reasonable also to assume that the maximum ped dimension x_m at the averaged depth $z = \bar{z}$ is equal to the distance $\bar{z}$ from the soil surface:

$$x_m(\bar{z}) = \bar{z} \tag{14}$$

(it does not mean, of course, that the largest dimension of an arbitrary ped at $z = \bar{z}$, i.e. x, is necessarily oriented along the vertical direction). From Eqs (12)-(14) and Eq. (6) one finds that:

$$d(z) = z_o(z/z_o)^{\bar{\omega}} \tag{15}$$

where

$$\bar{\omega} = 1 - 2\ln 2/\ln(\bar{z}/z_o), \qquad (0 < \bar{\omega} < 1) \tag{16}$$

For typical values $\bar{\omega}$=0.3 and $z_o \approx$30-60 cm, $d(z)$ increases rapidly with depth only in the first few millimeters. The condition, $\bar{\omega} > 0$ means that the average distance, $d(z)$ must increase with depth, and by accounting for Eq. (16) gives the relation between the parameters $\bar{z}$ and z_o (see Fig. 1):

$$\bar{z}/z_o > 4. \tag{17}$$

In order to express $\bar{z}$ through z_m (see Fig. 1) and to estimate the range of variation of the ratio z_m/z_o we use the following assumptions concerning the function $f_m(z)$: a) the depth $z = \bar{z}$ is at the middle of a transitional layer with most of the $f_m(z)$ variation (Fig. 1); and b) the half-width of the transitional layer at the depth $z = \bar{z}$ is equal to the mean ped dimension $d(\bar{z})$ at that depth (Fig. 1). With these assumptions the boundaries of the transitional layer are at $z_1 = \bar{z} - d(\bar{z})$ and $z_2 = \bar{z} + d(\bar{z})$, respectively. According to Eqs (6) and (14) $d(\bar{z}) = \bar{z}/4$. Moreover, the first assumption and Eq. (8) lead to $z_2 \cong z_m$. Hence:

$$z_1 = \bar{z} - d(\bar{z}) = 0.75\bar{z}, \quad z_m \cong \bar{z} + d(\bar{z}) = 1.25\bar{z},$$

$$\text{i.e., } \bar{z} \cong 0.8 z_m. \qquad (18)$$

Replacing $\bar{z}$ from Eq. (18) in Eq. (17) we get, besides Eqs (9) and (11), the inequality:

$$z_m / z_o > 5. \qquad (19)$$

In the range $0 \le z \le z_m$, $f_m(z)$ decreases from unity to zero (Fig. 1). With the above assumptions and Eq. (18) this may be written as follows:

$$1 - f_m(z) << 1 \quad \text{if} \quad z < 0.75z,$$

and

$$f_m(z) << 1 \quad \text{if} \quad z \to z_m \cong 1.25\bar{z}. \qquad (20)$$

It is assumed that either half of the transitional layer includes half of the variations of $f_m(z)$ from unity to zero (Fig. 1). Hence:

$$f_m(\bar{z}) = 0.5. \qquad (21)$$

Eqs (20), (21) and (17) lead to the simple approximation of the dimensionless function depending on the ratios z/z_o and $\bar{z}/z_o$:

$$f_m(z) = \left(1 + \exp\left((z - \bar{z})/z_o\right)\right)^{-1}. \qquad (22)$$

The connectedness, c as a function of soil depth, z results from Eqs (7) and (22):

$$c(z) = \ln\left(1 + \exp\left(-(z - \bar{z})/z_o\right)\right)/8.4. \qquad (23)$$

In Eqs (22) and (23) one may use $\bar{z}$ according to Eq. (18). The condition, $c(0) \le 1$ gives, according to Eq. (23), an estimate for the maximum of the parameter z_m/z_o, and with Eq. (19) we have:

$$5 < z_m / z_o <\cong 10.5. \qquad 24)$$

Using the extreme values of Eq. (24) and Eqs (22) and (23) we can find the range of values of c and f_m at the surface and at the boundary of the intensive-cracking layer, $z = z_o$:

$$0.48 <\cong c(0) \le 1 \text{ and } 0.36 <\cong c(z_o) <\cong 0.88 \qquad (25)$$

and

$$0.98 <\cong f_m(0) <\cong 1 \text{ and } 0.95 <\cong f_m(z_o) <\cong 1. \qquad (26)$$

According to Eqs (25) and (26) the values of the connectedness in this layer can be as small as 0.36, but they correspond to quite a large volumetric fraction of peds $(f_m \ge 0.95)$. Hence, the model is self-consistent. In the limiting case the maximum crack depth, z_m is assumed to be close to the water-table level. For the maximum value of one can consider $z_o \cong 0.1 z_m$ (Eq. (24)) because, in the limiting case, $c(0) \cong 1$. However, in any case, according to Eq. (11), we have $z_o \le z_s$.

GEOMETRICAL CHARACTERISTICS OF SHRINKAGE CRACKS

In the following the term "crack" means a vertical crack. The model was applied to estimate the width, the cross-section area, and the volume of the cracks. First the basic model [5, 6] was used to define the mean specific length of the crack traces per unit area of a horizontal cross-section, $L(z)$ (the function $N(x,d)$ is given by Eq. (2)):

$$L(z) = \int_0^{z_m} x N(x, d(z))\, dx =$$

$$\left[1 - \left(1 + z_m / d(z)\right)\exp\left(-z_m / d(z)\right)\right] / d(z). \qquad (27)$$

This specific length together with the vertical variation of the horizontal-surface shrinkage, $\delta(z)$ determines the expressions for the crack width at a depth z:

$$R(z,h) = \int_h^z L(z')^{-1} d\delta(z') \quad z < h \qquad (28)$$

(where h is crack-tip depth), for the cross-section area of cracks at the soil surface (the crack tips are between depths x and y):

$$A(x,y) = \int_y^x R(0,h) dL(h), \quad 0 \le x \le y \le z_m \qquad (29)$$

and for the specific crack volume (per unit volume of soil):

$$V(z,z_m) = \int_{z_m}^z R(z,h) dL(h). \qquad (30)$$

MATERIALS AND METHODS

To obtain model predictions of the crack width and specific volume in the micro-shrinkage depth range in swelling soils, as functions of soil depth, we used data on "depth - linear shrinkage" curves for seventeen field cases (El Abedine and Robinson [18] - eight cases; Yaalon and Kalmar [17] - six cases; Dasog *et al.* [9] - three cases). However, in these cases the "depth - linear shrinkage" curves are only in the macro-shrinkage depth range, $z < z_s$ as measured by a flexible wire of a given thickness (D=1.5 mm [17], 2 mm [9], 3 mm [18]). So we applied the following approach to estimate the linear shrinkage in the soil matrix, $\varepsilon(z)$ in the micro -shrinkage depth range $(z_s \le z \le z_m)$, based on the depths z_m, z_o, z_s and the diameter of a flexible wire, D. From the view point of measurements by a flexible wire [18] shrinkage below the depth z_s is absent. However, actual shrinkage of the horizontal surface at this depth, $\delta(z_s)$ can be estimated by:

$$\delta(z_s) \cong L(z_s) D \qquad (31)$$

where $L(z_s)$ is a specific length of crack traces on a horizontal cross-section (see Eq. (27)) at depth z_s. One can estimate the value $L(z_s)$ using the values z_m, z_o, z_s and Eqs (15), (16), (18) and (27). To estimate the corresponding linear shrinkage at depth $z_s, \varepsilon(z_s)$ one can use Eq. (31) and the relation:

$$\delta = \varepsilon(2 - \varepsilon). \qquad (32)$$

On account that the value of $\varepsilon(z_s)$ is very small (according to our preliminary estimates $(1 \div 4)10^{-3}$) we use a linear approximation for the dependence of ε on z in the micro-shrinkage depth range:

$$\varepsilon(z)=\varepsilon(z_s)\,(z_m-z)/(z_m-z_s), \qquad z_s \le z \le z_m. \qquad (33)$$

Values of z_m, z_o for eight profiles of El Abedine and Robinson [18] were found by least-square estimates of the mean crack width in the macro-shrinkage depth range. z_m and z_o do exist, they are of a single value and meet Eqs (9) and (11) and the relation for the maximum value $z_o \cong 0.1\ z_m$. The latter was used also for estimating z_o by z_m=225 cm of works [16,17], and z_m by $z_o \cong 76$ cm of work [9]. Values of z_m, z_o, z_s used for model predictions of crack width and specific volume in the micro-shrinkage depth range in all seventeen cases of works [9,17,18] are given in Table 1.

In the works [9,17,18] there are data on crack width and specific volume only for the macro-shrinkage depth range ($z<z_s$). So, model predictions, calculated with the values z_m, z_o, z_s from data of El Abedine and Robinson [18], Yaalon and Kalmar [17], Dasog *et al.* [9], were compared with data on specific volume and mean width of vertical cracks in the micro-shrinkage depth range from McKay *et al.* [13]. According to these data, in the micro-shrinkage depth range (1.5-5.5 m) porosity and width of vertical cracks ranges from $2\ 10^{-3}$ to $3\ 10^{-5}$to and from 43 to 1 μm, respectively.

RESULTS AND DISCUSSION

Figures 2 and 3 show, as an example, the predicted dependencies of crack width, R and crack porosity (specific volume), V on soil depth, z in the micro-shrinkage depths range for profile GHBTO7 from El Abedine and Robinson [18]. The dependencies $R(z)$ and $V(z)$ for the rest sixteen cases are similar with corresponding changes in values of z_s and z_m (Table 1). It may be seen that crack porosity predictions, in the micro-shrinkage depth

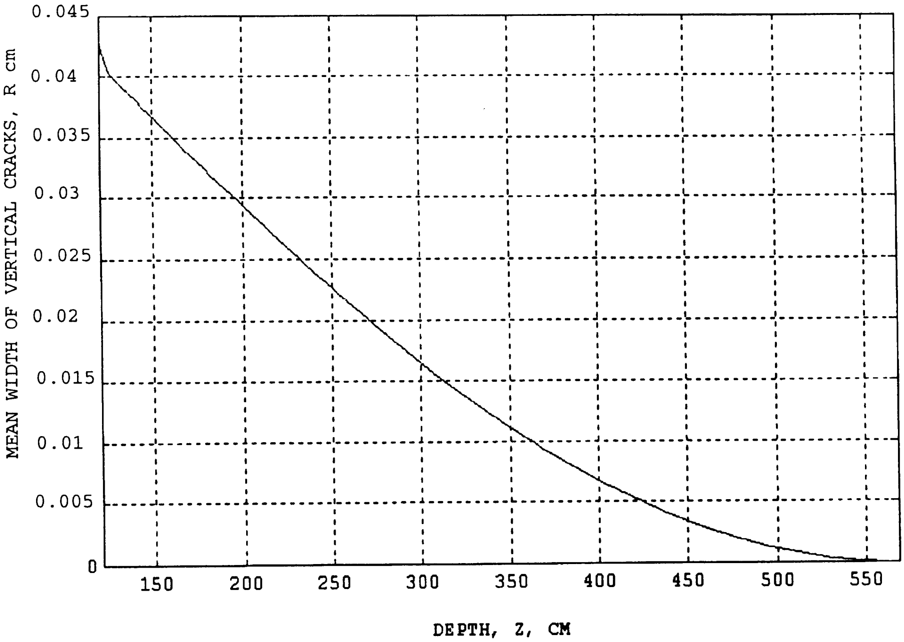

Fig. 2. Model predictions of dependency $R(z)$ in the micro-shrinkage depth range for profile GHBTO7 from El Abedine and Robinson [18].

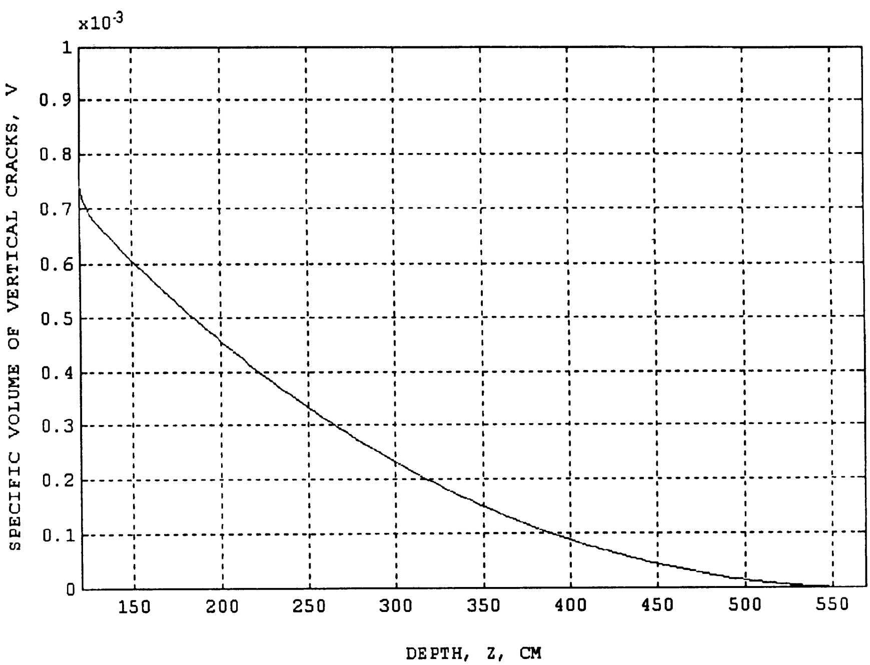

Fig. 3. As in Fig. 2 for dependency $V(z)$.

Table 1. Depth of macro-shrinkage layer z_s, model parameters, z_o, z_m and model predictions for crack width, R and porosity, V, at depth $z=(z_m+z_s)/2$ for seventeen cases

Case number	Reference number and profile designation or date	Measured depth of macro -shrinkage layer, z_s (cm)	Thickness of intensive-cracking layer, z_o (cm)	Maximum crack depth z_m (cm)	Crack width predictions for $z=(z_m+z_s)/2$, R (μm)	Crack porosity predictions for $z=(z_m+z_s)/2$, $V\ 10^4$
1	[18] GTO3	45	44*	524*	250	3.5
2	[18] GTO4	60	49*	540*	200	3
3	[18] GTO5	60	29*	360*	180	4
4	[18] GTO6	55	41*	420*	170	3
5	[18] O4H	75	36*	290*	75	2
6	[18] O4MH	75	37*	262*	55	1.5
7	[18] GHATO5	120	32*	293	70	1.8
8	[18] GHBTO7	120	61*	569*	120	1.5
9	[17] 30.05.74	30	21.4*	225**	90	3
10	[17] 20.06.74	40	21.4*	225**	70	2.5
11	[17] 16.07.74	80	21.4*	225**	45	1.5
12	[17] 26.08.74	70	21.4*	225**	50	1.6
13	[17] 19.09.74	80	21.4*	225**	45	1.5
14	[17] 21.10.74	80	21.4*	225**	45	1.5
15	[9] 26.07.84	80	76**	800*	130	1.3
16	[9] 09.08.84	100	76**	800*	115	1.2
17	[9] 06.09.84	120	76**	800*	105	1

*Calculated by least-square estimates or by the relation $z_o \cong 0.1\, z_m$. **Measured.

range, are within the limits of experimental values ($2 \cdot 10^{-3} \div 3 \cdot 10^{-5}$) [13]. However, crack width predictions are within such limits ($43 \div 1$ μm) [13] in only part of depth range $z_s \leq z \leq z_m$, adjacent to depth z_m. The reason for such behavior may be the following. In frame of the approximation by Eq. (31) estimates of R and V depend not only on the model parameters z_m, z_o and the experimental depth of the macro-shrinkage layer, z_s, but also on the diameter, D of the wire used in the experiment. The value of the latter reflects not the shrinkage cracking phenomenon in itself, but peculiarities of the measurement method in works [9,17,18]. However, at depths in distance limits of $\sim(z_m-z_s)/2$ to the depth z_m the "depth-linear shrinkage" curve given by Eq.(33) does not, practically, depend on D. Therefore at these depths ($\sim(z_m-z_s)/2 \leq z \leq z_m$) both V and R calculated by the model lie, for all the seventeen cases, in the limits of data from McKay *et al.* [13] or coincide with them in order of magnitude. Values of R and V at $z=(z_m-z_s)/2$ are given in Table 1.

CONCLUSION

The feasibility of the model approach to describe crack network geometry in swelling clay soil has been verified by showing agreement between the model predictions and published data on crack width and porosity in the micro-shrinkage depth range. Consequently it may be used as the basis for a model to describe hydraulic properties of vertisols.

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