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IMAGE ANALYSIS AS THE QUANTITATIVE ESTIMATION OF THE STRUCTURE OF AGRICULTURAL MATERIALS*

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A b s t r a c t. Deformable media which are the objects of interest of agrophysics are distinguished by the fact that they are composed of three phases and each of them clearly pronounce its specific role. Comparing existing theories of deformation and flow of three-phase media, at the very beginning, i.e., at the moment of definition and measurement of stress and strain one can recognize four methods: deterministic, statistical, probabilistic and stochastic method. The best is probabilistic method which is concerned with the formulation of the stress-strain response including microstructural effects that are due to the inherent geometrical and physical properties of structured solids. Most of the significant field quantities involved in the formulation of the material behavior are, by nature, random variables or functions of such variables. The method of computer image analysis together with a scanning electron microscope, scanning reflected light microscope and computer tomography are presented. Advantages of this method in comparison to the traditional methods of soil pore size distribution (mercury porosimetry, pF-curves and isotherm of nitrogen desorption) are demonstrated.

K e y w o r d s: probabilistic micromechanics, agricultural materials, structure, image analysis

INTRODUCTION

It is common that the effects of flow or deformation are measured on the surface of samples, blocks or profiles only. The structure and its changes are not taken into consideration and the nonlinearity inherent to the structure cannot be introduced into theoretical consideration by no means.

Deformable media which are objects of interest of agrophysics are distinguished by the fact that they are composed of three phases and each of them clearly pronounce its specific role:

- solid phase conserves the shape and strength;
- liquid phase is responsible for the effects of filtration, isotropic or spherical part of the strain tensor and dependence of processes on time;
- gas phase makes volumetric deformations possible, causing considerable changes of structure and strength during deformation.

Comparing existing theories of deformation and flow of three-phase media one can distinguish four methods at the very beginning, i.e., at the moment of definition and measurement of stress and strain (Table 1):

1. Traditional deterministic method is based on the assumption of homogeneity and continuity, stresses strains and flows are defined and measured in the form of derivatives and mechanical processes are considered as the motions in "ordinary" euclidean space x,y,z,t. [1,2,4,5]. As the consequence of the definition of stress and strain the linear thermodynamics of irreversible processes

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Table 1. Possible methods in mechanics of three-phase media

Deterministic method	continuous homogeneous medium, stresses and strain as derivatives: $\sigma=\frac{\partial F}{\partial S}, \varepsilon=\frac{\partial l}{\partial l_o}$, differential geometry motion in 'ordinary' space x, y, z, t	linear thermodynamics of irreversible processes, Onsager's reciprocal relations, linear, linear physical equations
Stochastic method	continuous, homogeneous medium measurable 'trend' ('mean value' and 'white noise')	classical equations of stochastic processes
Statistical method	discrete system of uniform elements (molecules, sphere, cells, cylinders, crystals), stress and strain as sums for all elements: $\sigma=\sum\frac{\Delta f}{\Delta S}, \varepsilon=\sum\frac{\Delta l}{l_o}$, motion in 'phase space' of coordinates and momentum (p, q, t)	statistical thermodynamics of irreversible processes, non-linear statistical physical equations
Probabilistic method	discrete system of unequal elements, stress and strain as integrals: $\sigma=\int d\sigma, \varepsilon=\int d\varepsilon$, 'motion' in space of experimentally obtained random variables	probabilistic physical equations formulated for one structural element (pore, grain, cell) and integrated for all random varaibles

together with the Onsager's reciprocal principle must be used for the formulation of constitutive relations. Some additional "latent" assumptions limit the validity of this method (flows should be laminar, potentials - parabolic, stresses, strains, gradients together with their derivatives - matematically infinitesimally small, state functions - differentiable and the system should be as close as possible to the thermodynamical equilibrium). As the result one gets unevoidably linear physical laws, which is far too rough approximation of real phenomena of deformation of such complicated media.

2. Methods of stochastic processes are utilized when one is not able to formulate any mechanism of the process being investigated. Experimental results of the process are compared to equations of specific stochastic processes (e.g., Markov process, Poisson process, random walk process). The discrepancy between measured and predicted values are discussed in terms of measurable trend (mean value) and white noise. This method has been applied in studies [1,5] for materials like steel and sand.
3. When the structure can be reasonably approximated by a system of equal elements (crystals,liquids, gases, spheres, cylinders, honeycombs) it is possible to use statistical thermodynamics. This method enables to define stress and strain as the sum (multiplication) of real effects for one structural element. In effect, being not to far from the state of thermodynamical equilibrium, one can formulate more realistic nonlinear physical laws (constitutive relations). Mechanical process is the motion in the "phase space" of generalized positions q, generalized momenta p and time t. We tried to apply this approach in studies [6,11-14].
4. In three-phase media with disorder, like agricultural materials composed as a discrete system of unequal structural elements (grains, aggregates, pores, cells, fibers) random variables or their functions are natural to play the role of independent walls variables of state, whereas the mechanical process should be a "motion" in the space of experimentally determined random variables and stress, strain or flow should be defined as integrals. Probabilistic equation for one structural element is to be integrated for all values of independent random state variables. We succeeded in formulation of such theory for soil deformation and its development for other agricultural materials is a matter of the nearest future.

The only one physically reasonable method of formulation of a probabilistic theory is to find random variables of state experimentally, i.e., to use a computer image analysis with a compatible microscope for the description of real structure and its changes during deformation.

STRUCTURE OF AGRICULTURAL MATERIALS AND RANDOM VARIABLES

To show the method of random variables we consider the soil compaction case. We will introduce now four random variables (Fig. 1), which together, with some deterministic variables, describe the structure of three-phase granular material and its changes. The probability densities of this random variables are:

- grain and aggregate size distribution $g_1(Ds)$, which describes the structure of a solid phase;
- pore maximum diameter distribution $g_3(Dp)$, deciding which soil grains or aggregates can enter into a given pore during deformation;
- pore volume distribution $g_4(Vp)$, informing about the soil volume which can enter into a pore considered;
- distribution of contact forces $g_2(f)$, responsible for the stress inhomogeneities.

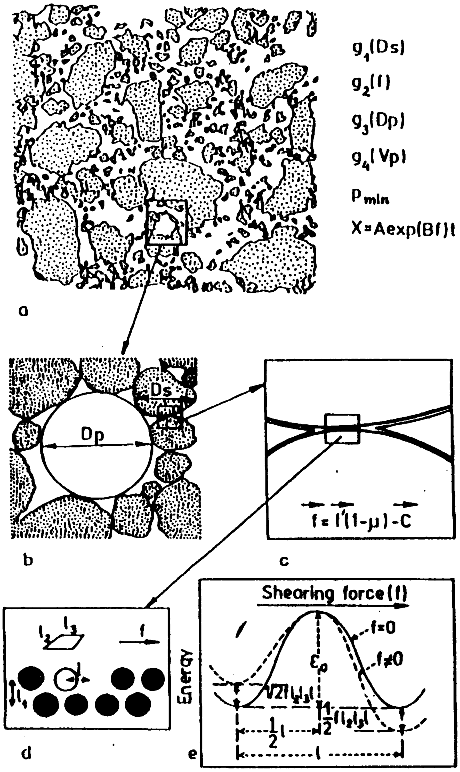

Fig. 1. Cross section through a sample of loess and qualitative description of its structure; a - image of sample cross section, b - pore cross section (Ds - grain diameter, Dp - pore diameter), c - contact of grains (f' - intergranular normal force), d - molecular layers (l, l_1, l_2, l_3 - intermolecular distances), ε_o - free energy.

The last effect can be seen for a model of granular medium in Fig. 2. The circles represent pills (cylinders) made of elastooptical material placed between two horizontal glass plates in a plane stress state. Forces between pills have been determined on the basis of the interference pattern in polarized light. The direction of lines between centers of circles represent the direction of forces, whereas their thickness - the value of this forces. One can see, how hardly this forces can be assumed parallel and equal.

As the soil consists of a great number of elements (pores, grains, aggregates) and interactions between them are extremaly complex it is natural to consider the parameters influencing on the soil deformation as random variables. In this case the complexity of the soil medium is an advantage which makes the consideration of its elements as statistical populations (sets, ensembles) possible.

In all practical agricultural problems the soil is relatively loose and at least 30 % of its volume is occupied by gas. Therefore the water is not squeezed out during deformation. To describe it quantitatively we will use random variables.

Let us estimate the density of random variable the values of which are the diameters of grains in the investigated soil. We will use the name "grain" in the meaning of elementary soil particle and aggregate unless the last is not destructed. We participate the set of values of diameters into intervals I_1, I_2, ...I_n. The participation follows from the used method of measurement of the grain size distribution and the greater number of intervals, the better description. We

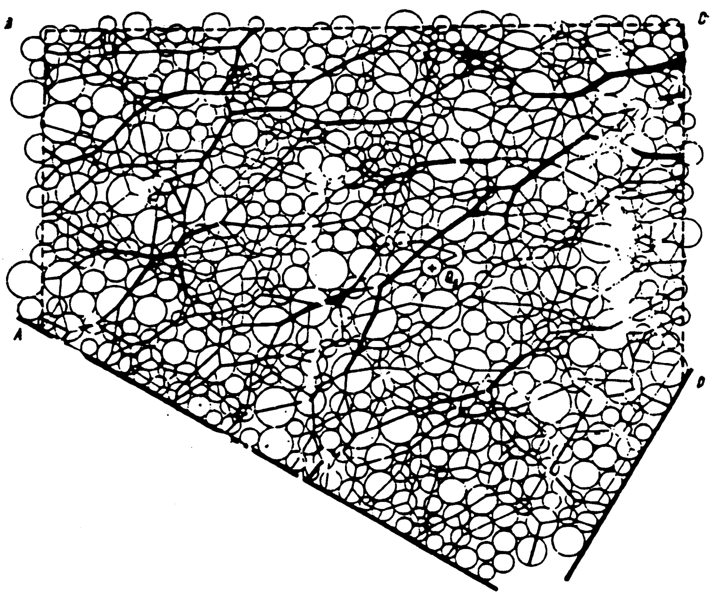

Fig. 2. Photoelastic visualization of intergranular forces [3].

can assume then, that the random variable is uniformly distributed in each interval and its density takes the following values:

$$g_1(Ds) = \sum_{k=1}^{n} \frac{N_k}{|I_k| N} J_{|I_k|}(Ds), \qquad (1)$$

where $|I_k|$ is the length of interval, N - the number of all grains and the indicator of set is:

$$J_{|I_k|}(Ds) = \begin{cases} 1, Ds \in I_k \\ 0, Ds \notin I_k \end{cases}. \qquad (2)$$

Analogously we will introduce the density of random variable representing the pore length for a fixed pore diameter:

$$g_3^{Dp}(l) = \sum_{j=1}^{m} \frac{N_j}{|II_j| N} J_{|II_j|}(l), \qquad (3)$$

and using the same notation the density of random variable representing the pore diameter:

$$g_4(Dp) = \sum_{i=1}^{l} \frac{N_i}{|II_j| N} J_{|II_j|}(Dp). \qquad (4)$$

Combining 3 and 4 one can get the two-parametric pore size distribution as the random vector:

$$g_5(Dp, l) = g_4(Dp)\, g_3^{Dp}(l). \qquad (5)$$

MOTION OF SOIL GRAINS INTO A FIXED PORE

Let F_o denotes the mean external force which is assumed to be the same in all directions for the simplicity. We will regard stress causing the viscous (time - dependent) motion of the grain as the difference between external stress f' and the internal friction $\mu f'$ together with cohesion c:

$$f = f'(1 - \mu) - c. \qquad (6)$$

Applying the theorem about the distribution of function of random variable the density of random variable representing t is:

$$g_2(f) = g'_2\left(\frac{f'}{1-\mu} - \frac{c}{1-\mu}\right) \frac{1}{|1-\mu|}. \qquad (7)$$

In the above formula g_2 is the Simpson's distribution for the sum of two independent random variables of uniform distribution. The density is symetrically concentrated around mean value and takes the values from 0 till $2f'_m$:

$$g'_2(f') = \begin{cases} 0 \text{ for } f' < 0, \\ 1/(f'_m)^2 \text{ for } 0 \le f' \le f'_m \\ -1/(f'_m)^2 (f' - f'_m) \text{ for } f'_m < f' \le 2f'_m \\ 0 \text{ for } f' > 2f'_m \ . \end{cases} \quad (8)$$

The mean intergranular stress has been calculated from the formula [9,10]:

$$f'_m = \left[\frac{F_o}{\frac{18}{\pi} \frac{1}{(EDs)^3 (1+e)}} \right]^{\frac{2}{3}} \frac{\pi a^2}{4}, \quad (9)$$

where e mean porosity index, a is the diameter of the mean contact area and ETD_s is the mean grain diameter.

From the consideration presented in [14, 15] we will assume the equation of motion of a soil grain in the form:

$$\Delta x = A \exp(Bf)\, t\,, \quad (10)$$

where Δx is the grain displacement and A, B represent the nonnewtonian viscosity dependent on temperature, water content, clay content and humus content. The lower boundary for the set of stresses causing the compaction is:

$$h\,(a, A, B, t) = \max \left\{ \frac{\ln a - \ln (At)}{B}, 0 \right\}. \quad (11)$$

The function $h(a, A, B, t)$ implies the fact that there exist t_{stop} such that the compaction stops independently on stress.

We can now estimate a new random variable the values of which are the diameters of particles entering into a pore. Assuming that:

- the grain cannot enter into a pore when its diameter is higher in value than that of the pore, i.e., a local increase of soil volume during compaction are neglected;
- the probability of entrance of grain into a pore is proportional to its surface or cross section, we can get the density of variable in the form:

$$g_6\,(Ds, Dp, L, f, t) = Dp\, l g_1\,(Ds)\, C_3\,(f_m)$$

$$\int_{h(a,A,B,t)}^{2f_m} g_2\,(f)\, df\,. \quad (12)$$

In order to express the final volume occupied by soil grains in the pore let us observe that the number of grains which are able to get into it is:

$$N\,(Dp, l) = \left[\frac{Vp}{V_{EDs}} (1 - p_{\min\, n}) \right], \quad (13)$$

where V_p is the volume of the pore, V_{EDs} - the volume of mean grain, $p_{\min}$ - the minimal porosity of the soil after compaction for a given stress f_m and the square bracket stands for the Entire function. It is obvious that the minimal porosity $p_{\min}$ decreases with the increasing stress f_m and it can be measured from the pore size distribution.

We have investigated the deformation of one pore. To construct the equation for all pores we have to take the initial volumes of all fraction from the experiment. V_{ok} denotes the initial total volume of all pores from the fraction k. After integration the following equation for the mean volume of grains entering into all pores in time t is obtained [14]:

$$V_t = \sum_{k=0}^{n} \frac{2}{3} \left[\frac{V_{pk}}{Ds} (1 - \rho_{\min}) \right] c_3(f_m)$$

$$\int_{h(a,A,B,t)}^{2f_m} g_2\,(f)\, df$$

$$\int_{Dp \in \Pi_\kappa} \frac{\int_{Ds_{\min}}^{\min(Dp,\, Ds_{\max})} (Ds)^3\, g_1\,(Ds)\, d(Ds)}{Dp}$$

$$g_4\,(Dp) \frac{N_p}{N_k}\,. \quad (14)$$

In this equation [ ]$=N(\overline{Dp, l})$ is the number of grains entering into the mean pore from the k - th fraction of pores (Eq. (13)). The difference between the total initial pore volume $\sum_k V_{ok}$ and the total final pore volume is the change of the sample volume V_t.

This equation gives the soil volume versus time for a constant external loading. It can be easily written in a three-dimensional vectorial or tensorial representation. It can be reduced to the theory of plasticity or visco-elasticity in some cases. This method enables to answer the following fundamental questions:

Which physical variables (deterministic and random) are responsible for the volumetric deformation of granular media?

Which are the mechanisms of this process and corresponding equations?

Why and how the quantity of bigger pores is decreasing during deformation abruptly?

Why the instantaneous deformation is almost completely irreversible?

How to formulate the same equations for all types, kinds and varieties of soil?

Similar considerations have been made for the deformation of plant cellular or fibrous material. In this case structure is described quantitatively by random variables the values of which are sizes related with the cell or fibre properties, respectively.

It is no accident then, that the classical mechanics has not been able to obtain a functional formulation of the physical relations (constitutive equations) and in order to obtain their linearity it has restricted itself to formulation the local relations (theory of plasticity or viscoelasticity, laws of energy or mass flow). To recognize the structure of the three-phase agricultural materials it is necessary to introduce an integral condition between forces and flows or deformations respectively, together with a quantitative measure of structure in the form of random variables.

DETERMINATION OF POROSITY AND CONCLUSION

Comprehensive comparatory investigations of different methods of the pore size distribution determination have been conducted by Lawrance [7], Lawrance *et al.* [8] (Fig. 2) and Bouma [3] (Fig. 3). As it can be seen in Fig. 3 the maximum found for the Compton Beauchamp clay in the range 30 to 100 nm probably indicates that the dominant structural units are domains, i.e., small clusters of clay particles of about 1μm size. Between 100 and 1000 nm the porosity is greater after critical point drying (dotted line) than indicated by the water content - suction curves. One can conclude that this difference represents the pores which collapse during drying and release water at pF below 3.5. The fact that the apparent porosities obtained from the water content - suction curves are greater than those of the CP dryied samples in the range 1000 to 100000 nm accord with such conclusion. The results lead to a conclusion that all the methods of determination of pore size distribution except image analysis are limited to a range, within which each of them gives a reasonable values.

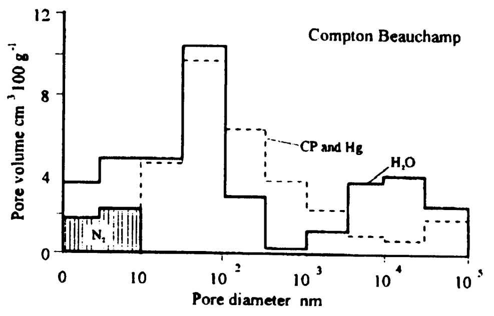

Fig. 3. Comparison of different methods of pore size determination (Lawrence [7,8]).

Bouma [3] (Fig. 4) showed that only wetting water suction - water content curve can pretend to fit with micromorfological method of the pore size distribution curve determination.

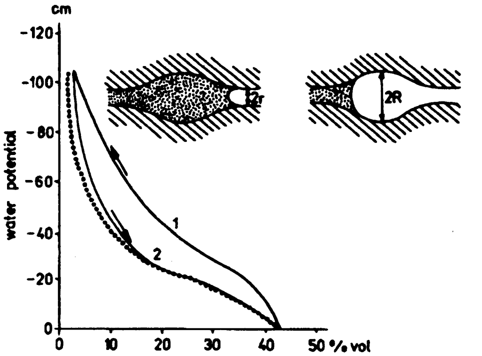

Fig. 4. Comparison of pF - curve pore size determination with the image analysis (dotted line) [3].

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