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STEREOLOGY AS AN IMAGE ANALYSIS METHOD OF AGRICULTURAL MATERIALS*

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A b s t r a c t. The present study is concerned with the application of the methods of stereological analysis to the investigation of the structure of agricultural material. The most important issue in the investigation of such three-phase media is to account for their structure, which is continually changing in the deformation process, and more precisely the physical parameters of the structure such as the size of solid phase elements and pores being a complementary phase. The study presents examples of the structures of agricultural materials (soil, root, grains, potato tuber cells) and their changes resulting from the operation of external forces. Common features of the materials being investigated and resulting search for a common method to investigate their structure have been stressed here. Determination of the distribution of grain size, aggregates, pores and forces in the contact of the solid phase elements and the sizes of cells, cell walls and their strength parameters is indispensable while applying a probabilistic equation to determine the changes of medium volume at any time during the deformation process. A probabilistic equation for a three-phase, granular medium (soil) considering for the function of the distribution of structural element sizes has been presented as an example. In order to solve the problem of empirical determination of such functions a stereological analysis has been proposed. Basic assumptions of such approach have been presented stressing their use for the quantitative characteristics of the solid phase element geometry as well as of the geometry of the space created by those elements and porosity being the substructure of the material being investigated. Using stereological analysis one can quantitatively determine physical parameters describing the three-dimensional structure of the object being investigated on the basis of the analysis of a flat image. As it is easy to obtain microscopic images of agricultural materials, it seems futuristic to learn such approach. Stereological analysis has not, so far, been applied in the investigations of agricultural materials (except for own investigations of the authors). That is why the present study includes the basic information and references.

K e y w o r d s: stereology, image analysis, agricultural materials

INTRODUCTION

One of the most important physical features of the agricultural materials being investigated is their structure. There exists a number of the definitions of a structure but all of them practically determine it a spatial system of solid phase elements and a system of pores [14].

The structure of agricultural materials undergoes continual changes as a result of various mechanical interactions, which cannot be avoided in contemporary, mechanised agriculture. The problems concerning soil cultivation, crop collection and transport, as well as the entire processing of crops obtained require us to foresee the deformations and damages of agricultural media as they lead to substantial loss in crops and agricultural production.

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To this end it is indispensable to know the basic dependence stress-strain-time, to determine which the classical methods of rheology or theory of plasticity are not enough as they do not account for the physical parameters of the agricultural medium being investigated. The structure of agricultural materials cannot be compared with simplified, uniform elements like cylinders, balls, etc.

Quickly developing measurement techniques, particularly in the field of micromorphological research show that the strain processes - shear strain, volumetric strain, cracking - have a similar course in numerous agricultural media being investigated. The common feature of these processes is the dependence on the physical properties of a medium being investigated and changes in its structure.

A medium reacts to the operating stress by a change in its structure and its subsequent states decide about further physical processes connected with the displacement of liquids and gases and heat flow, as well as mechanical strength. The construction of the structure is influenced by physical features of individual elements, their geometrical configuration, the connections between them, density, porosity, surface area, elasticity, hardness, etc.

Previous detailed theoretical and experimental research of the authors on the common mechanics of agricultural materials applying a probabilistic approach allowed to foresee the changes in medium volume against time during deformation considering the function of the distribution of the size of structural elements [13,17].

Our research on the mechanics of three-phase agricultural media required the search for methods, which allow for quantitative description of the structure of materials being investigated. So far, in our research on the experimental determination of the function of the distribution of structure elements we have obtained promising results using stereological analysis [7,10].

In the subsequent part of the study we present examples of structures and their changes resulting from the operation of forces for soil, root, grain (in mass) and potato.

Common properties of such three-phase media and a method of their determination as the quantitative characteristics of the solid phase element geometry as well as of the geometry of the space created by those elements and porosity being the substructure of the investigated material.

Using stereological analysis one can quantitatively determine physical parameters describing the three-dimensional structure of the object being investigated on the basis of the analysis of a flat image.

SOIL

Microscopic observations of soil structure, especially its changes caused by different cultivation exertions, indicate that external forces may bring about both quantity and aggregates shape changes. These forces may also cause aggregate disintegration and crushing, aggregate blocks formation of large density formed from smaller aggregates, and the pores filling solid phase can not be compared to capillary or other geometrical shapes (Fig. 1 and 2 [9]).

The compound elements of soil structure have different quantity and irregular shape. They are connected between one another in a fairly permanent way. Even small external forces cause changes in such a structure; it is at the cost of gas phase changes - porosity. In the process of deformation, the number of solid phase particles contact points into a section unit changes. This causes changes of all mechanical quantities. Particle movement of such a medium is disordered; irreversible deformations predominate over the reversible ones. Moreover, the basic - stress - deformation - time dependence can not be recorded by means of finite number of deterministic equations. Because of assumptions deterministic and statistical thermodynamics methods turned out to be unuseful for real deformation process description [16,17,20].

Because of great particle aggregates and pore numbers and their quantity, shape and configuration varieties, probability methods- first applied in soil science - are a correct ways of structure analysis [19].

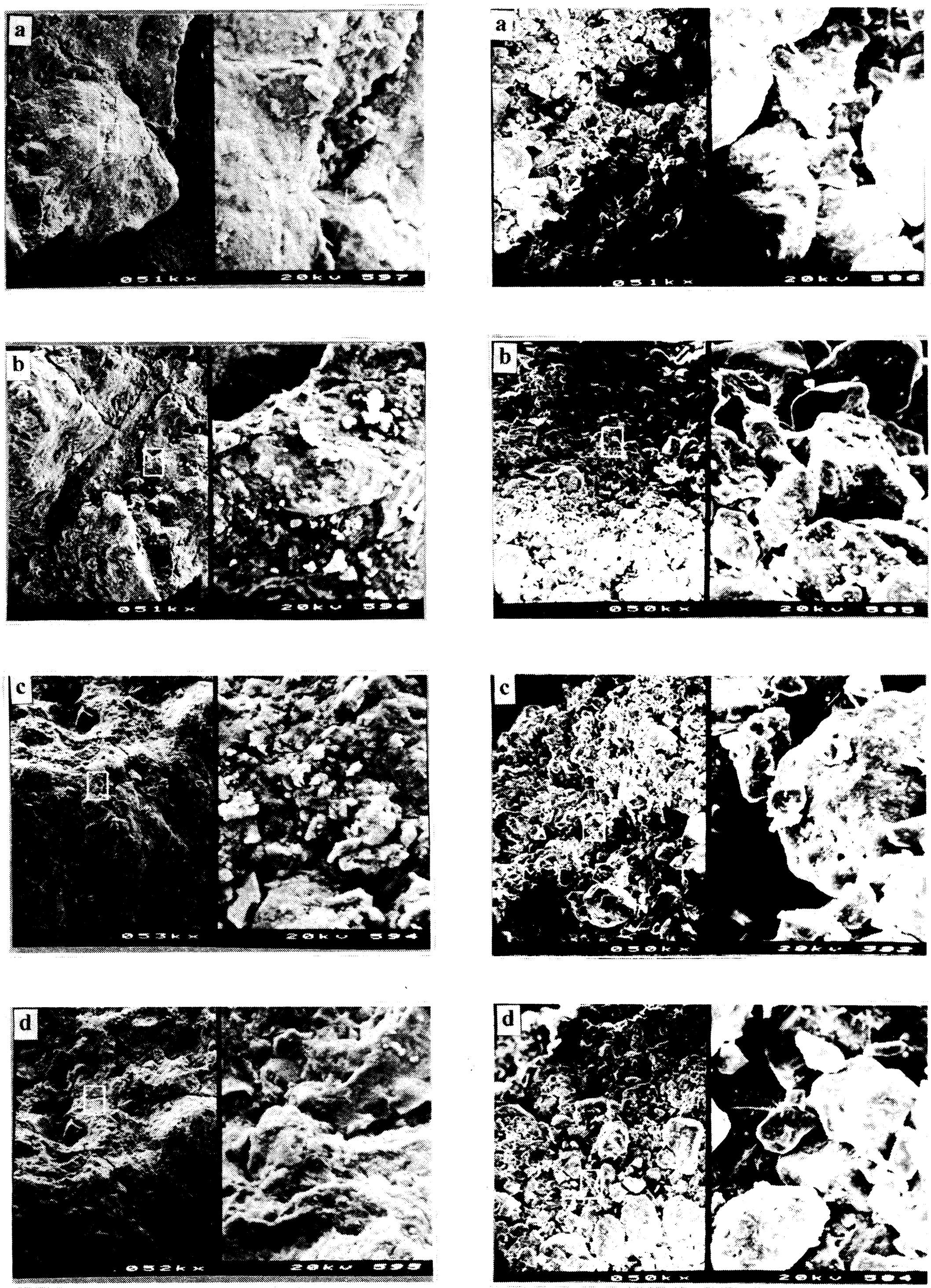

Fig. 1. Microscopic photos (SEM) of loess soil structure - the left picture and sand soil - the right picture: a - test, b - plough, c - cultivator, d - harrow; at two magnifications x50 and x500.

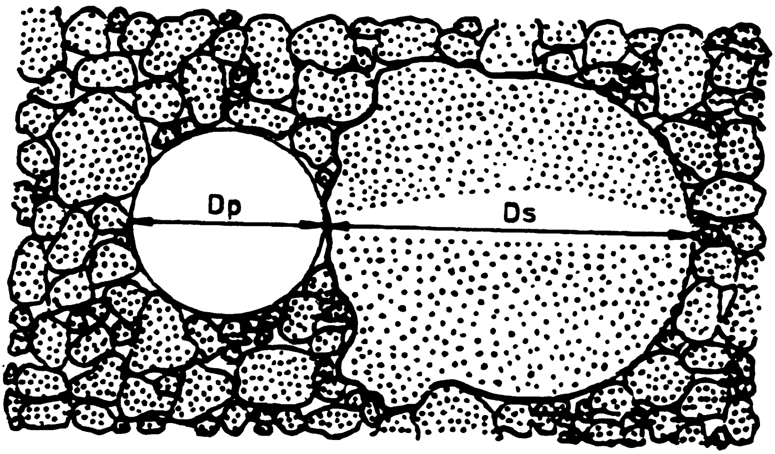

Fig. 2. Soil particle of the diameter bigger than that of pore, can not displace itself into its inside without the pore increasement.

These methods where also applied in soil deformation examination obtaining a new, totally original soil volumetric deformation equation including medium structure [21,22]:

$$V_t = \sum_{k=0}^{n} V_{0k}\frac{2}{3}\left[\frac{EV_k}{ED_s}(1-p_{\min})\right]C_3(f_m)$$

$$\int_{h(a,A,B,t)}^{2f_m} g_2(f)df \times$$

$$\int_{D_p\in\pi_k} \frac{\int_{D_{s_{\min}}}^{\min(D_p, D_{s_{\max}})} D_s^3 g_1(ds)dD_s}{D_p} g_3(D_p)\frac{N_p}{N_k}$$

where V_t - pore volume change versus time, D_s - particle diameter, V_{ok} - initial k fraction volume, EV_k - initial k fraction pores average volume, ED_s - average partical diameter, f_m - mean contact stress, $p_{\min}$ - minimal final porosity, $g_1(D_s)$ - solid phase partical quantity distribution, $g_2(f)$ - stress distribution in partical contact points, $g_3(D_p)$ - maximal pores diameters distribution, $Ds_{\min}$ - minimal partical diameter, min $(D_p, D_{s_{\max}})$ - a condition that a particle moving to a pore must be smaller than its maximum diameter of a pore, (Fig. 2), $C_3(f_m)$ - normalisation constant.

Random variables g_1, g_2 and g_3 play an important role in the quantitative estimation of soil structural changes during deformation. They cannot be obtained using traditional methods except by image analysis [12].

Because of unstable soil structure, the researches on this structure require sample preparation. After filling soil pores with a liquid substance (glue, resins) and hardening it, one may obtain sample intersection (Fig. 3, [7]). Bright and dark areas properly referring to porous space and soil solid phase constitute irregular areas with the opposite phase elements. Phase borders have a developed area, sometimes of low contrast, difficult to unique determination. Binary images obtained from image analysers increase contrast between the phases and eliminate problems of observed areas quantitative estimation.

Connection of soil structure changes with external forces that cause these changes, e.g., mechanical interactions, requires quantitative determinations. Choice of the measured parameters and kind of measurement from taking samples to image analysis becomes the main problem of this paper.

ROOT

Root and plant system development follows in the soil medium. It is directly connected with the physical soil condition (Fig. 4, [18]). Soil

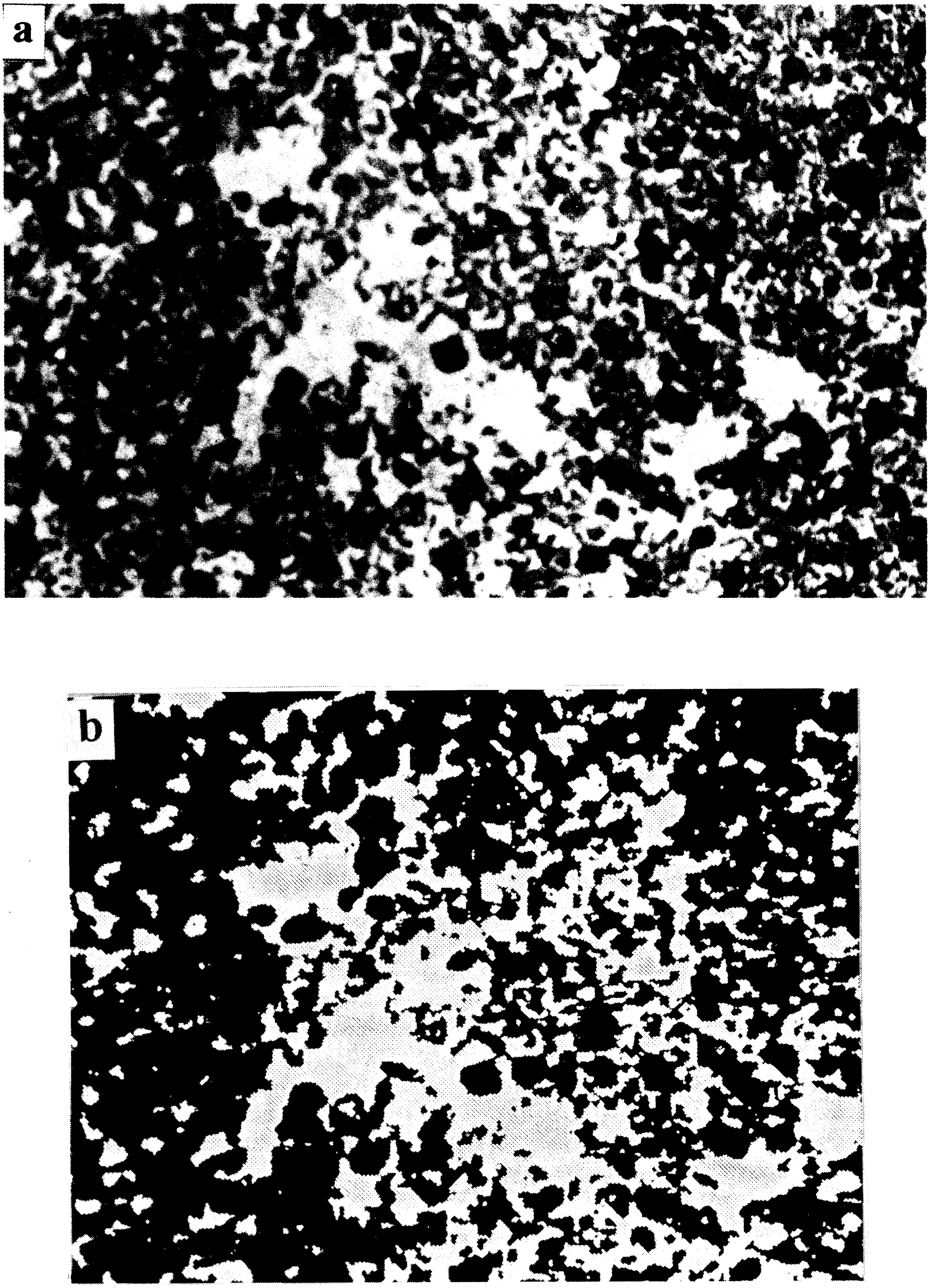

Fig. 3. The loess soil structure - the picture of the polished section from the microscope with the magnification of x350 and identification of structure obtained by means of automatic image analyser [7].

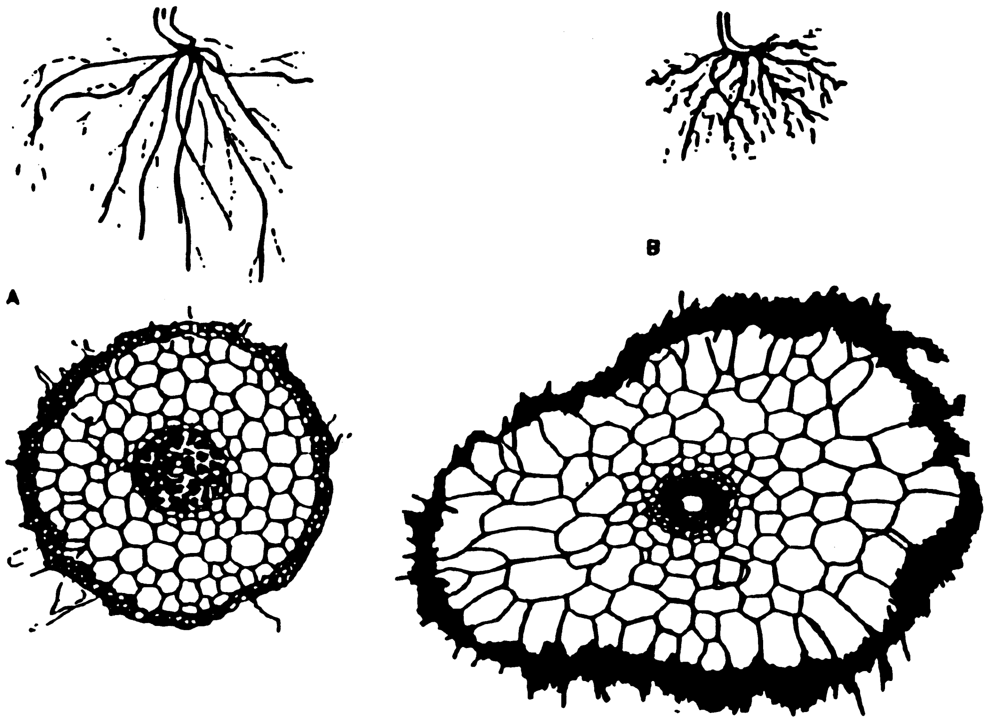

Fig. 4. The root system of spring barley in the loess soil profile and their intersections. A - looes soil, B - condensed soil [18].

density increasement causes the increase of penetration resistance and as a result - root length decrease. Such a dependence occurs in different depths in soil profile and remains through the whole vegetative period. It was proved by field experiments of different plants, soils and different soil loads.

The changes of soil physical properties influence not only the root length but also its structure. The crosswise spring barley root and lengthwise corn root intersections indicate changes in internal cells shapes and development of compact areas between them. Despite cell deformation there occur air spaces and necrosis areas (Fig. 5 and 6 [1]).

The structures that consist of such heterogeneous elements regarding contacts between unitary elements and the reasons causing the changes, require quantitative description.

GRAINS

Grain arrangement in a silo, a medium of distinct anisotropic features, is the next example of different structure formation (Fig. 7 [15]). The analysis of sample medium intersection picture, made in different planes, shows distinctive unspherical grain cubic orientation and porosity.

Grain piling up in a definite and controlled way leads to concrete structures which decide about the way of the total mass behaviour - both at grain outflow and at the influence on the container walls and its bottom.

The knowledge of anisotropy and physical elements features of such a medium allows for forseeing its mechanical characteristics.

The forces, acting on the grain in the silo may also cause surface grain damage and complicate interactions between the whole structure (internal friction).

POTATO

In the case of other media such as, e.g., potatoes, we deal with piling up into heaps and clumbs. A component element size distribution is big and random. The forces produced a b

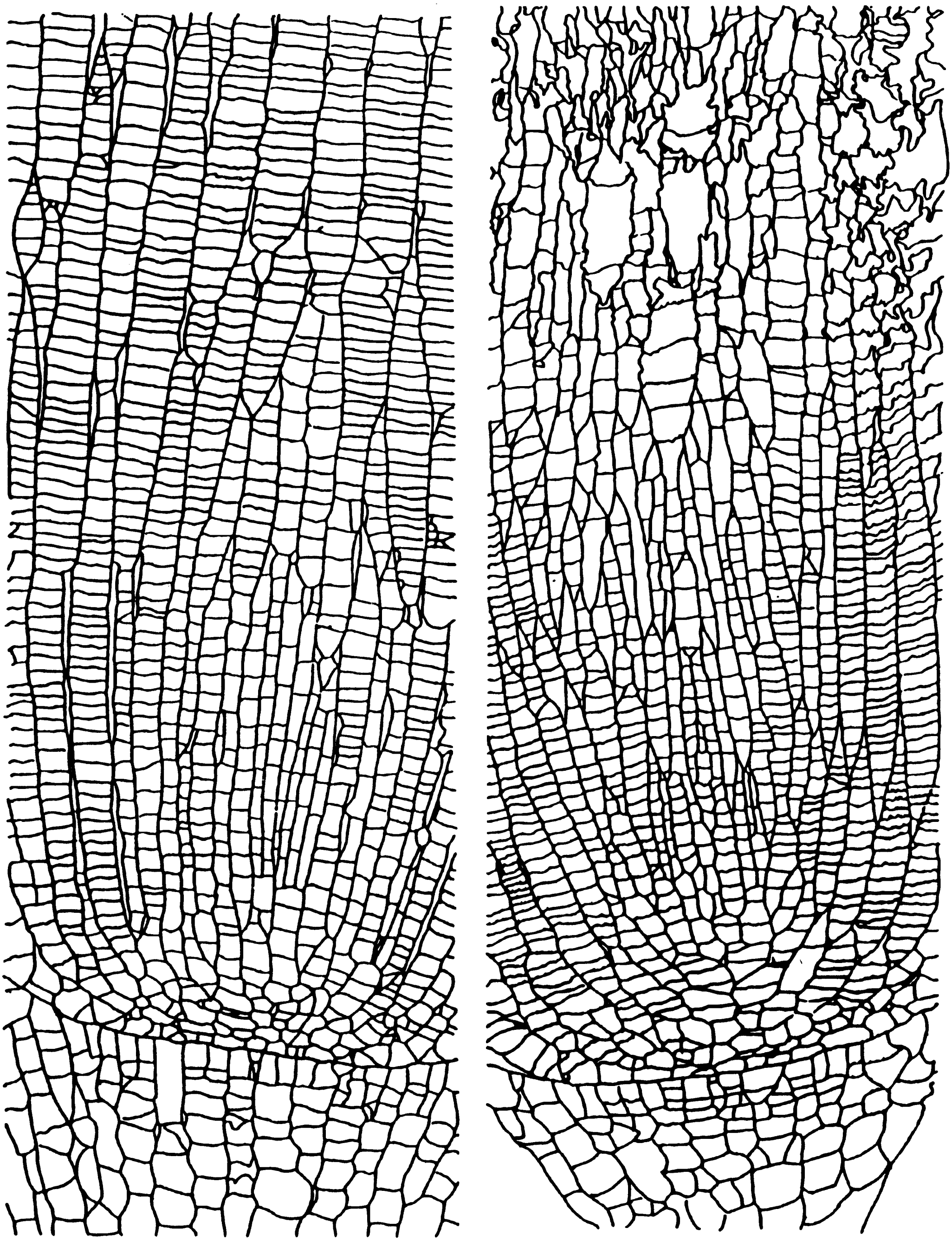

Fig. 5. The image of oblong corn root intersections with the magnification x400. a - loose soil, b - condensed soil [J. Lipiec - unpublished].

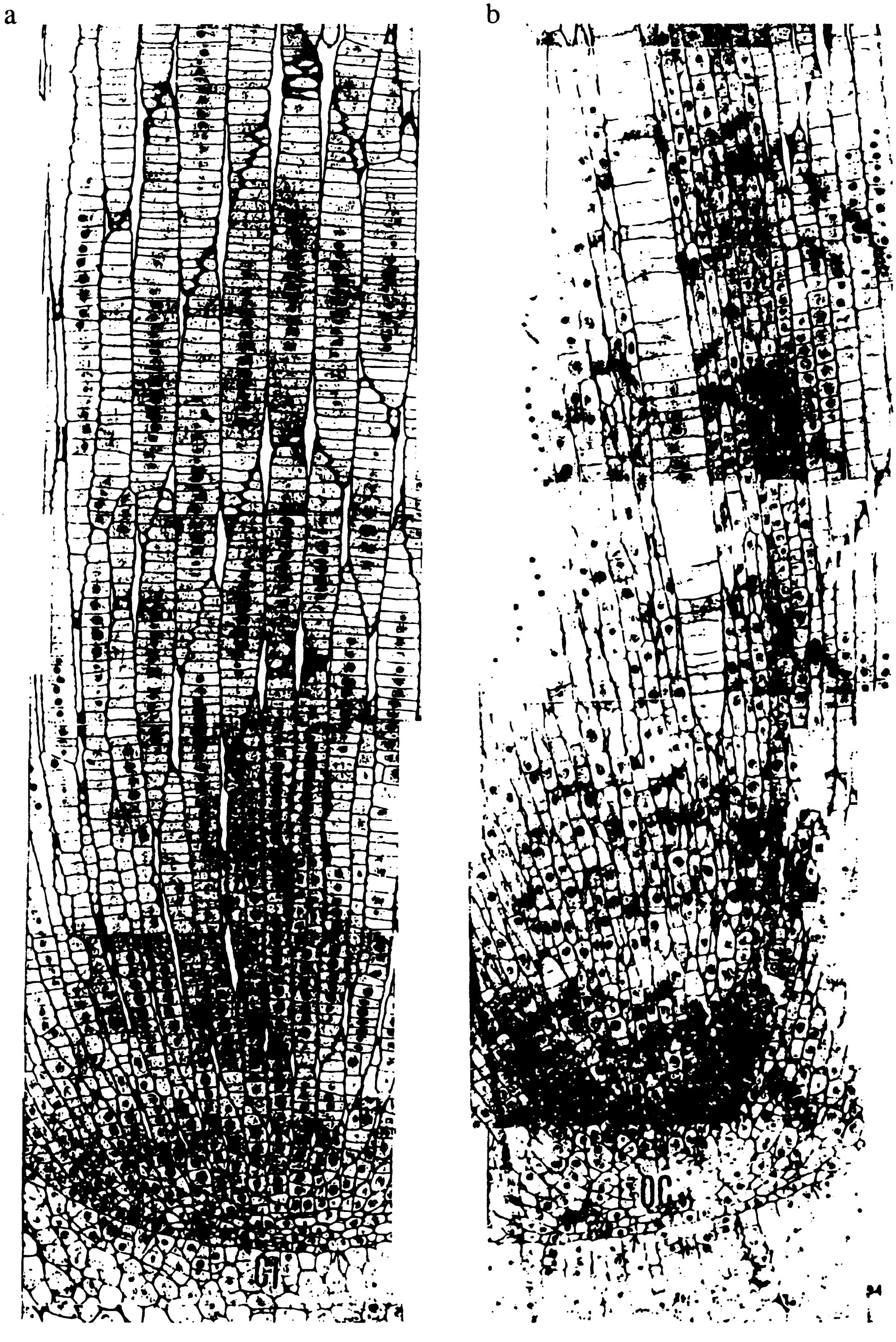

Fig. 6. Oblong corn root intersections in conditions: a - oxygen free, b - oxygen. Visible air-spaces between cells [1].

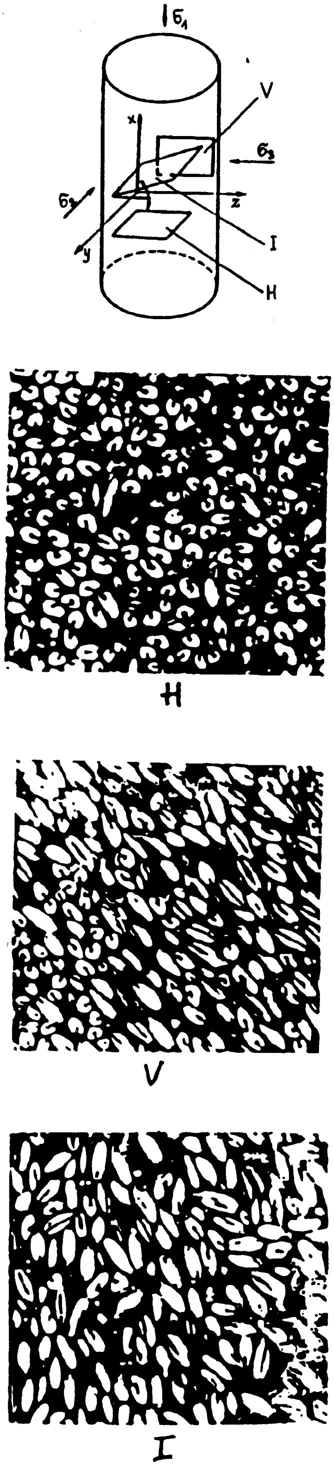

Fig. 7. Intersection photos of cylindrical sample of rye grain. Intersections obtained in 3 different planes H, V, I, [15].

during the storage and transport are considerable and lead to a bulb damage. The damages are not always visible. They often occur internally, on the cell level and only in the further stage they lead to crop loss. Microscopic analysis allows for identification of potato sample cell structure (Fig. 8 and 9 [12]).

Confocal scanning reflected light microscope enables observations on unprepared samples in real time. It allows for cell structure changes recording at the presence of external forces and numerical processing of the obtained discontinuous images of the examined process course (e.g., cracking).

PROBLEMS OF THE EXAMINED STRUCTURE QUNTITATIVE DESCRIPTION

All the examined agricultural material structures are characterised by:
- discreet homogenous structure,
- random character of the physical processes that occur in them.

The examined structures constantly change in time, and in practice we deal with several instantaneous states. These states decide about the repeated courses of physical processes. For the quantitative description of such structures, it is necessary to introduce probability approach together with the whole procedure of physical parameters determinations as random variable.

In the case of soil loose media it is necessary to determine grain size, aggregates, pores and forces in solid phase element contacts distribution. In the case of fibrous and cell structures - cell sizes, cell walls, fibres and their resistance parameters.

STEREOLOGICAL ANALYSIS OBJECT

The previous chapters imply that the following individual objects can be the subject of geometrical characteristics: plant cells, crop grains and clumbs, soil particles and their aggregates. In the stereological analysis method description, the individual objects that occur individually or form different structures will be called - bodies or grains. They will be treated as convex geometrical

Fig. 8. Fine details of potato cell walls shape in optical scannning microscope. Magnification x500 [12].

Fig. 9. Cross section of potato tissue in optical scanning microscope. Magnification x500 - a, x1000 - b [12].

objects. The main object of stereological analysis is the quantitative characteristics of:

- individual grain geometry,
- geometrical, three dimensional structure of compact material, formed by a definite grain collections.

Agricultural materials can be built of one or several component grains. These form the so called material skeleton. The skeleton does not always fill the 3D area that is enclosed by material or its sample. Pores are the filling components. They can form different space configurations. There are two configurations that are most frequently distinguished: isolated pore system and connected pore system. The material porosity forms one of its possible substructures.

Traditional grain size analysis was concerned with characterizing geometrical features of individual grains or grain collections that created loose material. The modern grain size analysis also includes quantitative estimation of compact material space structure. Stereological analysis is one of the grain size analysis method. Depending on the kind of the solving problems, stereological analysis is the main relatively adventive completing grain size analysis method. Agrophysics development opens new areas for stereological analysis applications and may influence its progress.

Stereological analysis applies two methods: a projection method (i.e., of shadows) and an intersection method. The individual grain geometrical characteristics is mostly based on the projection method, i.e., collection information about grains by their shadows geometrical features measurement (of width, areas and circumferences). Because of the lack of adequate measurement devices, the projection method has not been widely applied so far.

In the intersection method we can distinguish planar, linear and point analysis. The method is applied for characterizing material geometrical structures, and exceptionally for individual grain geometry. The information source about space structure material is a geometrical mosaic of its traces on one or many intersections (thin microsections, cutting, polished sections etc.), made from the material samples. The collected pieces of information mostly concern grain intersection area, its circumferences length and a number. The linear analysis is based on the fact that regular straight line lattice is led on sample material intersection. Then, we measure chord lengths formed by lines with grain cut and count the chords. The point analysis is the random or regular point distribution on the sample material intersection. We count the points falling into the grain cuts that belong to individual components. The information which is obtained from the planar, linear or point analysis is the basis for some parameters quantity estimation which characterize the space material structure.

The essence of the method is that we conclude about the structure features from space structure geometrical traces that are visible on intersections, lines and points. It is obvious that the information obtained this way is much more scanty than the information that could be obtained by direct space structure presentation. That is the reason why the stereogical analysis is applied at the moment when we cannot make any space penetration without damaging the material structure itself.

The stereological analysis rudiments reach half of the XIX century. Then, Dellse - the French mineralogy scientist noticed that the volume proportions of the individual minerals forming a rock are the same as their microsections sections area proportions. From the beginning of the 1960, we notice the connection of stereological and image analysis method development. The available automatic devices possess an ample software and allow for efficient stereological analysis of simple structures. But the proper analysis understanding must be based on its mathematical fundamentals.

BASIC FUNCTIONALS OF CONVEX BODIES

Let G be a geometrical body. If for every couple of points $P_1, P_2 \in G$ segment $P_1, P_2 \in G$, then G is called convex body. Ellipsoids, rectangular prisms and cylinders are the examples of convex bodies. A stake is the example of a non-convex body. The functional of convex body is the $X(G)$ number assigned to this body. $X(G)$ functional is called the basic functional in Hadwiger's sense [11] if it possesses invariance property considering movement group in space additivity and monotonous. The following examples are the basic functionals: volume $V(G)$ $[l^3]$, area $S(G)$ $[l^2]$, total average curvature $M(G)$ $[l^1]$, total Gaussian curvature $C(G)[l^0]$.

The polyhedron total average curvature is determined by the formula:

$$M=\frac{1}{2}\sum_i l_i k_i, \tag{1}$$

where l_i - i-th polyhedron edge length, k_i - angle formed by normals to the walls crossing along i-th edge.

The total average curvature of smooth body limited by the formula:

$$M=\frac{1}{2}\int_s\int\left(\frac{1}{R_1}+\frac{1}{R_2}\right)dS, \tag{2}$$

where R_1, R_2 - main curvature radii in the point of body area.

The total cubicoid curvature is equal: $M=\Pi(l_1+l_2+l_3)$, for the sphere, $M=2\Pi D$. The total convex body average curvature has simple geometrical interpretation. If D_{Fe} determines the so called Feret's [3], body diameter, i.e., its average width, then the following relationship proceeds:

$$M=2\pi D_{Fe}, \tag{3}$$

in which the factor of proportionality does not depend on body in the convex body class. The polyhedron total Gaussian curvature [11] is determined by the formula:

$$C=\sum_i \varepsilon_i, \tag{4}$$

where ε_i - space angle formed by the normals to the walls converging in i-th corner. The total Gaussian curvature of the smooth body of C^2 class surface is determined by the formula:

$$C=\int_s\int\frac{1}{R_1R_2}dS. \tag{5}$$

The total Gaussian curvature of every convex body is constant and equals $C=4\Pi$.

Up to the Hadwiger's theorem [11], every basic functional of convex G body can be presented as follows:

$$X(G)=aC(G)+bM(G)+cS(G)+dV(G) \tag{6}$$

but the factors $a, b, c, d,\geq 0$ do not disappear simultaneously; they are uniquely assigned to the body.

The convex body class also includes the so called degenerated bodies. The following ones belong to them:

- common part of $GE\neq 0$ body G and of E plane, i.e., smooth convex figure. Its basic funcionals are:

$$V(GE)=0,\quad S(GE)=2s(GE),$$

$$M(GE)=(\pi/2)l(GE),\quad C(GE)=4\pi \tag{7}$$

where $S(GE)$ and $l(GE)$ are: area and circumference of the smooth figure GE.

- common part of $GL\neq 0$ of body G and of L straight line is a segment. The basic segment functionals are:

$$V(GL)=0,\quad S(GL)=0,$$

$$M(GL)=\pi t(GL),\quad C(GL)=4\pi, \tag{8}$$

where $t(GL)$ stands for segment length.

- common part $GA\neq 0$ of body G and point A is point. Its basic functionals are:

$$V(GA)=0,\quad S(GA)=0,$$

$$M(GA)=0,\quad C(GA)=4\pi. \tag{9}$$

REMARKS ON STEREOLOGICAL ANALYSIS BASIS

As it was mentioned before, stereological analysis is based on projections and intersections method. The projection method mathematical bases are the so called Cauchy's projection formulae [11]:

$$M=\frac{1}{2}\int b(n)dn,\qquad M=\frac{1}{2\pi}\int l(n)dn,$$

$$S=\frac{1}{\pi}\int \sigma(n)dn. \tag{10}$$

Their forms are important for a convex body. In the formulae (10) - $b(n)$ stands for G body width to n direction, i.e., distance between the planes with normal vector n planes that support *body* G from up and down; $l(n)$ and $\sigma(n)$ adequately for circumference length and body G perpendicular area into the plane of normal vector n; dn indicates a differential of solid angle spread out around direction n. Integration spreads over into the whole 4Π solid angle. By means of the formulae (10) only M (in two ways) and S can be determined. The projection does not provide a relationship for determining body volume. This functional should be determined by other methods.

Crofton's formulae [2,11] are the mathematical basis of the intersection method. Lets start with the planar analysis. We will consider geometrical information included in the set of all common parts $GE \neq 0$ of body G and of plane E . The formulae (7) show that three of the basic functionals SE are not identically equal 0. They constitute a basis for deriving the first three Crofton's formulae:

$$M = \int_{\geq\neq} dE, \quad S = \frac{2}{\pi^2} \int_{GE\neq 0} l(GE)dE,$$

$$V = \frac{1}{2\pi} \int_{GE\neq 0} S(GE)dE . \qquad (11)$$

The differential $dE = dndp$, where dn stands for the elementary three solid angle, spreads around normal vector n to plane E. dp is elementary displacement of the plane E movement along this vector. Integration spreads over all the plane E positions with $GE \neq 0$. The formulae (11) indicate that we can determine three functionals of our body G from the information obtained on flat intersections (number of sections, their circumferences lengths and their area). In the linear analysis we consider the information included in the set of common parts $GL \neq 0$ body G and straight line L. The formulae (8) show that we can base on two functionals, and then we obtain the next two Crofton's formulae:

$$F = \frac{2}{\pi} \int_{GL\neq 0} dL, \quad V = \frac{1}{2\pi} \int_{GL\neq 0} t(GL)dL . \qquad (12)$$

The differential $dL = dndp$, where dn is the elementary solid angle around directional vector n of straight line L. dq is the elementary area determined by straight line L on the plane that is perpendicular to the line. Integration spreads over all the straight line L positions in the space with $GL \neq 0$. In the point analysis we consider information included in the common part of body G and point A. This is point A. The formulae (9) show that we have only one functional of positive value. Then we obtain the last Crofton's formula:

$$V = \int_{GA\neq 0} dA . \qquad (13)$$

The differential $dA = dpdq$, and presents elementary volume determined by the point A. The integration spreads over all the point A positions with $GA \neq 0$. By means of point analysis we can only determine the volume of body G. On the integration ground, probabilistic sense can be added to the above, introduced differentials dE, dL, dA. They determine the principle called: plane; straight line and point respectively isotropic uniform random distribution (IUR density). Such of interpretation differentials in the formulae (11)-(13) gives the way to the introduction of the concept of unknown estimators $[V,S,M]$ of body G - that is the introduction of mathematical statistics - in a practical situation, when we do not have full sets GE, GL, GA but we have only their finite number and we cannot directly use the formulae (11)-(13). Let's assume that we have two convex bodies G and G_0; but body G is included in G_0. Apart from it, the position of body G is optional. Let's ask what is the probability of cutting body G by the plane E under the condition that the plane crosses the body G_0; that is symbolically signed: $Pr\{ GE \neq 0 | G_0E \neq 0\}$. Analogical question can be put forward as for hitting body G by straight line L or point A. Considering the principle of random isotropic density of planes, straight lines and points in space, i.e., relying on some of the Crofton's formulae (11)-(13) we obtain:

$$Pr\ \{\ GE \neq 0 \mid G_0E \neq 0\} = \frac{M}{M_0}, \quad (14)$$

$$Pr\ \{\ GL \neq 0 \mid G_0L \neq 0\} = \frac{S}{S_0}, \quad (15)$$

$$Pr\ \{\ GA \neq 0 \mid G_0A \neq 0\} = \frac{V}{V_0}. \quad (16)$$

Being aware of these simple relationship one can prevent from introduction of intersection distribution principles in material samples by the simultaneous application of estimators based on the formulae (11)-(13). This is the essence of Bertrand's paradox.

INDIVIDUAL GRAIN GEOMETRICAL CHARACTERISTICS

Size and shape are the basic space attributes of an individual grain. The statement that the class of geometrically congruent grains is the set of grains of the same size, and another statement that the class of geometrically similar grains is the set of grains of the same shape - express intuitive understanding of size and shape grain concept. Together with technology development, there increases the need of size and shape quantitative estimation introduction for individual grain belonging to wider set than the similar grain class.

Only in some cases the grain name uniquely informs about grain shape, e.g., a sphere or a cube. But for cuboid shape characteristic it is necessary to give proportions of edge length. Here we will present the proposal formed in the paper [5] that concerned the introduction of convex grain shape and size natural characteristics based on the basic grain funcionals. In this proposal, grain size stands not for one but for three numbers:

$$[V, S, M] \text{ or } [V, S, D_{Fe}], \quad (17)$$

where D_{Fe} stands for Feret's diameter (see Eq. (3)). Consequently the so called Blaschke's coefficient couple should be considered as grain shape quantitative characteristics [10]:

$$x = 4\pi \frac{S}{M^2} = \frac{1}{\pi} \frac{S}{D_{Fe}^2},$$

$$y = 48\pi^2 \frac{V}{M^3} = \frac{6}{\pi} \frac{V}{D_{Fe}^2} \quad (18)$$

satisfying the inequalities:

$$0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le y \le x^2. \quad (19)$$

Here we will not consider all the grain shape and natural size characteristic properties. We will only present two favourable points:

- characteristic is based on basical grain functionals,
- characteristic easily allows for noticing all the difficulties faced at determining grain shape and size notions.

These definitions lack mutual unambiguity consisting in the fact that if exactly one triple of numbers (17) corresponds to every convex grain and, in consequence, one pai of Blaschke's coefficients (18) then exactly one grain, infinitely many grains, and even no convex grain at all may correspond to the selected triple of positive numbers (17). Hadwiger [10] mentions characteristic example of cone and cylinder - the bodies of different shapes but of the same triple value [17]. We can put forward the agreement that by means of finite parameters number we cannot introduce unique shape and size grain definitions in the sense above suggested.

Geometry deals with problems even of the individual body of unique characteristics. It uniquely characterizes them within the same similarity class. Analogical situation occurs in the case of material structure quantitative geometrical estimation. The unique geometrical structure characteristics can be given within the same similarity class.

In the natural shape and size grain characteristics there are 3 functionals [V,S,M] of convex grain. Relying on projection method, Cauchy's formulae give the bases for determining only S and M. Volume V must be determined by means of another method. Unfortunately, the projection method faces practical application difficulties because of the devices shortage

[4,8]. In specific situations, when we have a congruent grain number we can apply intersection method, and Crafton's formulae estimators (11)-(13) for determining the above mentioned functionals triple. But then, we should additionally know the so called 'specific grain number'.

CHARACTERISTICS OF GRAINED MATERIAL GEOMETRICAL STRUCTURE

Material can be made of one or many components grains.The consideration will be limited to the 2-component material. Soil can be the example of such a material if the solid phase particles are considered as one component, whereas pores as the second component. It may happen that in the multiple material analysis (e.g., polymineral rock), we are only interested in the structure formed by grains of one component. Then, all the other components are treated as the so called 'matrix'. Let's consider the sample of material that fills 3D region R of volume V_R. Let's determine by (V_i, S_i, M_i,) i= 1,...,N, volume, surface area and total mean curvature of i-th grain that is in the region R. Quantities described by the formulae:

$$V_v=\frac{\sum_i V_i}{V_R},\quad S_v=\frac{\sum_i S_i}{V_R},$$

$$M_v=\frac{\sum_i M_i}{V_R},\quad N_v=\frac{N}{V_R} \qquad (20)$$

are the set of basic parameters that characterize 2 component material structure but the second component is treated as so called 'matrix'. We call them as: V_v - volumetric ratio or specific volume, S_v - specific surface area, M_v - specific total mean curvature, N_v - specific quantity of the chosen component.

The word 'specific' informs that the defined parameters are mean value per volume unit of the sample R of a material. Basing oneself on the intersection method, i.e., on Crofton's formulae set (11)-(13) - stereological analysis gives estimators of unknown parameters V_v, S_v, M_v. They are compiled in the Table 1. The set of four parameters (20) is considered as grained materials quantitative structure basic characteristics. Crofton's formulae (11)-(13) do not give any basis for determining estimator of N_v. So far it has been impossible to derive an estimator of this parameter assuming grain convexity on the basis of the information obtained on non correlated intersections.

Total characteristics knowledge (Eq. (20)) allows for derivative characteristics introduction. A set of derivative parameters:

$$\left[\frac{V_v}{N_v}, \frac{S_v}{N_v}, \frac{M_v}{N_v}, 1\right] \qquad (21)$$

presents volume, surface area, and total mean curvature of the chosen grain component on the average per grain. The next derivative parameters set in relation to (20) is the set:

$$\left[1, \frac{S_v}{V_v}, \frac{M_v}{V_v}, \frac{N_v}{V_v}\right] \qquad (22)$$

which presents total mean curvature and the number of the given component grains on the average per volume unit of this component. It is characteristic that the sets of (21) and (22) parameters are independent of packing of grains of the chosen component in the space.

It is worth mentioning that sets of (20) and (22) parameters show other aspects of space structure although they are based on the same information sets.

CONCLUSION

Agrophysics is more and more interested in stereological analysis. Generally, these are the attempts of the verification of method usefulness already applied in rock mechanics or in biology, and also looking for new solutions for specific agrophysics problems.

Only the introduction into stereological analysis geometrical basis was presented above. The considerations were limited to the convex bodies as elements of geometrical structure. For example estimators V_v, S_v, compiled in

Table 1. A basic parameters estimators that characterize the grained material structure

Size	Definition	Estimators from stereological analysis		
		planar	linear	point
Specific volume V_v	$\frac{\Sigma V_i}{V_R}\frac{l^3}{l^3}$	$\frac{\Sigma S_i}{S}\frac{l^2}{l^2}$	$\frac{\Sigma t_i}{t}\frac{l^1}{l^1}$	$\frac{z}{Z}\frac{l^0}{l^0}$
Specific surface area S_v	$\frac{\Sigma S_i}{V_R}\frac{l^2}{l^3}$	$\frac{4}{\Pi}\frac{\Sigma L_i}{S}\frac{l^1}{l^2}$	$4\frac{n}{t}\frac{l^0}{l^1}$	
Total specific mean curvature M_v	$\frac{\Sigma M_i}{V_R}\frac{l^1}{l^3}$	$2\Pi\frac{m}{S}\frac{l^0}{l^2}$		
Specific quantity N_v	$\frac{\Sigma N_i}{V_R}\frac{l^0}{l^3}$			

Explanation: ΣV_i, ΣS_i, ΣM_i, N - total: volume, surface area, total mean curvature and grains number in material sample of volume V_R, ΣS_i, Σl_i, m - total: area, sircumferences length and number of grain sections in the surface area intersections of S, Σt_i, n - total: chords length and their number on measurement line of t length; z, Z - number of points hitting the component and total points number.

Table 1 preserve their importance for non-convex bodies.

The structure estimation can be considered in another way. Structure characteristics (number, function) is 'the answer' of the structure itself for the 'analysing element'. So far, we applied the following analyzing elements: plane, straight line, point. There can be other:planes, straight lines, points couples or a segment. So, the information of geometrical structure do not have to be limited to the set of common structure parts and plane, straight line or point. The information can be also derived from the neighbouring of plane, straight-line or a point.

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