Int. Agrophysics, 1994, 8, 277-282

NORMAL AND TANGENTIAL STRESSES IN THE FRICTIONAL CONTACT AREA BETWEEN WHEAT GRAIN AND A FLAT SURFACE*

M. Molenda, J. Horabik

Institute of Agrophysics, Polish Academy of Sciences, Doświadczalna 4, 20-236 Lublin, Poland

Abstract. Measurements of friction force of a pair of wheat grains against a flat plate sandwiched between them were performed. Five levels of plate roughness and five levels of the grain moisture content were adopted. Two models were applied to describe the friction force as a function of normal load: linear Coulomb's law of friction and two element non-linear equation developed from molecular-mechanical theory of friction. Both surface roughness and grain moisture content were found to influence model parameters. The Coulomb law of friction was found satisfactory for design calculations. Non-linear model appeared more accurate, particularly in the range of lower normal load. Moreover, the non-linear model parameters have been shown to be more closely related to the physical phenomena involved in the process of friction.

Keywords: wheat grain, friction force, friction coefficient

EXPERIMENTS

Measurements of the contact area were made by applying the technique of microphotography in reflected light. The grain was placed on the microscope table and loaded through a smooth glass plate. The image of the contact area was photographed in reflected light. Measurements of the friction coefficient were performed on pair of wheat grains sliding against a plate sandwiched between them. Details of the experimental procedure may be found in the reports [1,2].

The examinations were conducted on Grana variety wheat grain of the moisture content 8 %, 11 %, 13 %, 15 % and 18 % wet basis. Both contact area and friction force measurements were conducted under the same conditions of loading. Seventeen values of normal loading were adopted within the range from 0.029 N to 4.30 N (effective pressure in grain mass from about 0.0017 MPa to about 0.26 MPa). The experiments were performed under constant room conditions to minimize the influence of the ambient temperature and relative humidity. Each variant of the experiment was repeated ten times.

The height of roughness - R_t (DIN 4762) of the sliding plate was measured with an optical profilometer. The R_t of smooth glass plate and polished steel plate was below 0.5 μm, the lower measuring range of the profilometer. The other steel plates were machined and the values of R_t equal 1.0 μm, 1.8 μm, 6.4 μm and 11 μm were obtained.

RESULTS AND DISCUSSION

The widely accepted model of the process of friction is the Coulomb law of friction [1]. The relationship between the normal force and tangential force (i.e., the force of friction) takes the form of:

$$T = C + \mu N \qquad (1)$$

where T is the tangential force, C is the cohesive force, is the friction coefficient, and N is the normal force. The above formula is commonly used for the design calculation

---

*This work was supported by the State Committee for Scientific Research, Poland under grant No. 5 S306 030 04.

involving the process of friction as proposed by Coulomb in 1781. Since the Coulomb law of friction does not provide a physical interpretation of the process, later investigators contributed many theoretical or experimental models to the understanding of the process of friction.

Examinations of contact micro-area by Bowden and Tabor [3] pointed to the possibility of separation of the frictional resistance into two components: adhesive and deformational. Similar description of the friction process was given by Kragelsky [4], who claims that molecular interaction takes place in the layer one hundredth micrometer thick, while deformational interactions - in the layer above one tenth of micrometer. Therefore, the resulting frictional resistance can, in the first approximation, be expressed as the arithmetical sum of molecular and deformational forces.

Kragelsky [4] proposed the expression for the coefficient of friction:

$$\mu = \frac{\tau_0}{p_r} + k_x + k_1 \alpha \left(\frac{h}{r}\right)^{\frac{1}{2}} \qquad (2)$$

where τ_0 - shear strength of adhesive bond, p_r -normal stress, k_x - coefficient comprising the influence of normal load on the shear resistance, k_1 - coefficient comprising the type of deformation in the contact area, α - coefficient expressing action of the hysteresis, h/r - geometrical parameter. The first two elements of the expression constitute the adhesive component and the third one represents the deformational component of the coefficient of friction. Taking the definition $\mu = T/N$, and $p_r = N/S_r$ one get:

$$T = \tau_0 S_r + N \left(k_x + k_1 \alpha \left(\frac{h}{r}\right)^{\frac{1}{2}}\right) \qquad (3)$$

The first component of the expression depends on the true contact area while the second component is the linear function of the normal force. The true contact area may be expressed according to the Hertz formula $S_r = A_1 N^B$. Denoting the expression in parenthesis with a letter *a*, an exponent *B* with a letter *c*, taking $b = \tau_0 A_1$ and substituting one obtains:

$$T = aN + bN^c \qquad (4)$$

The first component of this expression is analogous to the second component of Coulomb's Eq. (1) and represents the deformational component of the force of friction. The second component of the Eq. (4), which is the power function of the normal force, as in Coulomb's, approach comprises 'the attraction of the surfaces' and represents the adhesive component of the force of friction. When describing the process of friction with the Eq. (4) the value of the force of friction for zero of normal load is equal to zero, while the force of cohesion *C* in Coulomb's formula, which is ambiguous in interpretation, does not appear. Moreover, calculation of the coefficient of friction from the definition $\mu = T/N$ does not result in unnaturally high values of μ in the neighbourhood of N=0.

Both Eqs (1) and (4) were fitted to the experimental data. The values of the obtained parameters with their statistical estimates are presented in Table 1 for Eq. (1) and in Table 2 for Eq. (3).

Coulomb's law of friction

Both moisture content of grain, *w*, and surface roughness significantly influence the values of coefficient of friction, μ, and the cohesion, *C*. The influence of surface roughness is stronger than that of the grain moisture content. Figure 1 represents the influence of grain moisture content on the coefficient of friction for five levels of the height of surface roughness, R_t. In the case of smooth surface the relationship between the coefficient of friction and grain moisture content, $\mu(w)$, exhibits the minimum for 15 % of grain moisture content and may be approximated by parabolic curve. In the case of the surface of R_t=1 μm, coefficient of friction μ - increases permanently

Table 1. Parameters of the model: $T=C+\mu N$ estimated by linear regression procedure (wheat grain cv. Grana, sliding against a steel plate)

Rt (µm)	Grain m.c. (%)	μ	C (mN)	Corr. coeff., (%)
<0.5	8	0.088	4	98.6
	11	0.073	6	98.5
	13	0.076	6	99.2
	15	0.062	5	98.1
	18	0.111	9	98.9
1.0	8	0.197	4	99.7
	11	0.209	6	99.7
	13	0.209	8	99.7
	15	0.224	-7	99.7
	18	0.239	-8	98.8
1.8	8	0.300	-2	99.6
	11	0.339	9	98.7
	13	0.296	-7	99.7
	15	0.297	25	99.5
	18	0.410	-22	99.7
6.4	8	0.547	-23	99.2
	11	0.535	-10	99.0
	13	0.566	-20	98.7
	15	0.526	20	99.0
	18	0.548	-15	99.7
11.0	8	0.258	-12	98.8
	11	0.328	-30	98.3
	13	0.351	-31	98.6
	15	0.379	-44	98.5
	18	0.386	-50	98.5

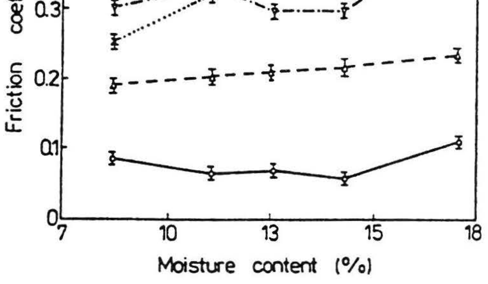

Fig. 1. Effect of moisture content on coefficient of friction for five levels of the height of roughness, R_t (cv. Grana wheat grain sliding on the steel plate).

with the increase of grain moisture content and the approximation with the linear equation may be applied. No clear tendency of the $\mu(w)$ relationship for the other surface roughness cases was noticeable. Changes in the $\mu(w)$ relationship probably reflect the changes of the phenomena involving friction resistance in various cases of surface roughness. In the case of the smooth plate, when the effect of adhesion is the dominant, the relationship with a minimum is observed. With increasing surface roughness, the role of deformational forces increases. For the height of roughness $R_t=6.4$ µm, the interactions on the chaotically dispersed surface asperities result in the large scatter of experimental data and the lack of significant differences among means of the coefficient of friction. Only in the case of the smooth surface all the values of cohesive force, *C*, take the positive sign (Table 1). With the increase in surface roughness growing number of means *C* takes the negative sign. This effect reflects the change of the shape of the relationship, *T(N)*, between the friction force and normal load.

Kragelsky's equation

The values of the coefficient of determination, R^2, for the curvilinear model (Table 2) are higher than the squared values of the coefficient of correlation, *R*, for the linear Coulomb model. The analysis of variance indicated a significant increase of the model parameters *a* and *b* with the increase of the surface roughness. The parameter *c* increases significantly with, the R_t increasing in the range from 0 µm to 1 µm and above that level does not change significantly. The exponent *c* takes the values from 0.65 (close to Hertz's calculations) up to 1.65. The change of the exponent *c* reflects the change in the shape of *T(N)* relationship, from convex ($c<1$), through linear ($c=1$) to concave ($c>1$). In the case of the approximation of the *T(N)* relationship with the Coulomb formula the change in shape of the experimental data plot was reflected in the change of sign and value of the cohesive force, *C*.

Table 2. Parameters of the model $T=aN+bN^c$ estimated by non-linear regression procedure (wheat grain cv. Grana, sliding against a steel plate)

Rt (μm)	Grain m.c. (%)	Model parameters a	Model parameters b	Model parameters c	R^2 (%)
<0.5	8	-0.002±0.450	0.095±0.450	0.92±0.36	99.3
	11	0.044±0.034	0.040±0.035	0.78±0.21	99.4
	13	0.050±0.060	0.370±0.061	0.75±0.45	97.2
	15	0.044±0.030	0.026±0.031	0.71±0.37	98.0
	18	0.085±0.030	0.041±0.031	0.68±0.27	98.9
1.0	8	0.104±0.464	0.104±0.464	0.91±0.40	99.7
	11	-0.440±1.663	0.268±1.663	0.95±0.32	99.7
	13	0.109±0.293	0.115±0.293	0.89±0.29	99.7
	15	0.104±1.784	0.110±1.784	1.04±0.75	99.6
	18	0.105±31.80	0.128±31.80	1.01±3.34	98.5
1.8	8	0.180±4.260	0.120±4.260	1.04±1.30	99.5
	11	0.080±2.760	0.290±2.760	0.93±0.71	98.4
	13	0.240±0.140	0.040±0.140	1.22±0.63	99.7
	15	-0.170±0.960	0.519±0.950	0.92±0.15	99.7
	18	-0.055±1.850	0.320±1.850	1.06±0.34	99.6
6.4	8	0.296±0.950	0.208±0.950	1.14±0.57	99.1
	11	0.232±45.40	0.292±45.40	1.02±2.70	98.7
	13	0.370±0.960	0.163±0.960	1.16±0.89	98.5
	15	0.116±0.890	0.466±0.890	0.87±0.26	99.1
	18	0.322±2.810	0.206±2.820	1.06±0.82	99.5
11.0	8	0.102±0.970	0.138±0.970	1.10±0.68	98.7
	11	0.179±0.085	0.088±0.085	1.50±0.37	98.3
	13	0.100±0.270	0.192±0.270	1.25±0.31	98.6
	15	0.186±0.126	0.108±0.123	1.42±0.36	98.4
	18	0.219±0.061	0.069±0.056	1.65±0.33	98.6

Figure 2 represents the typical plot of experimental data together with the both approximating lines for wheat grain (cv. Grana) of 11 % moisture content (w.b.) sliding on the smooth steel plate. Both approximations show high degree of correlation. From the statistical point of view, linear model is the better one because incorporating the non-linearity makes the equation more complex and does not improve fitting of the model to the experimental data. However, as shown on the graph in the neighbourhood of the origin the straight line diverges markedly from the experimental data, while the curve adheres expressly to the experimental points. Thus Kragelsky's model describes the process of friction more accurately than the linear Coulomb formula.

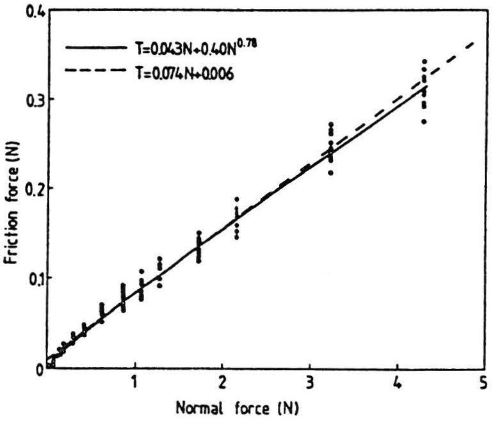

Fig. 2. Force of friction in relation to normal load. Course of experimental data and the two models fitted (cv. Grana wheat of 11 % moisture content sliding on the smooth steel surface).

It seems that only in the case of the smooth surface the friction force may be separated into adhesive and deformational components. For more complex distribution of surface asperities (in the case of higher roughness) adhesive forces act on the contact areas randomly located in the space and the simple separation of the two components is not possible. The coefficients of determination in Table 2 are high, and so are high standard errors of the parameters. Because the three-parameter model was relatively unstable, the constant value of the exponent c=0.75 was taken for further calculations. The procedure of multiple regression was applied to fit the equation $T = aN + bN^{0.75}$ to the experimental data. The estimated parameters are presented in Table 3. The table contains also the values of the *b*/*a* which represents the ratio of the adhesive component of the friction force to the deformational component. The *b*/*a* values in Table 3 are close to those reported by Kragelsky *et al.* [4] for polythene or teflon ($b/a<1$). The ratio *b*/*a* is higher for the steel surface than that for the glass surface for all grain moisture content levels tested. It may be concluded that the unit adhesive strength is higher for the steel than for the glass. Taking the expression for the adhesive component of the friction force - $T_a = \tau_0 S_r$ and the results of earlier experiments on the true contact area S_r, the unit adhesive strength may be calculated from the formula: $\tau_0 = bN^{0.75}/AN^B$.

The results shown in Fig. 3 demonstrate that the unit adhesive strength τ_0 is independent on the normal load and equals about 1 N/mm^2. The results demonstrate also the tendency of τ_0 to decrease with the increase of grain moisture content.

The results obtained agree with Buckley's interpretation of the process of friction [5], who claimed that elastic deformation appears at first on the contact area, and with increasing normal load the role of plastic deformation increases. The state of equilibrium establishes when the contact area is large enough to support the load. The force of friction is related to the contact area, i.e., the larger the contact area is the higher is the friction resistance on it. Friction forces between the surfaces of solid bodies depend on chemical and physical states of the surfaces in contact. The chemical composition of the atmosphere of environment is also significant because the surface films may change the adhesive properties of the materials in contact.

Table 3. Parameters of the model: $T=aN + bN^{0.75}$ estimated by multiple regression procedure (wheat grain cv. Grana, sliding against steel and glass plates)

Grain m.c. (%)	Model parameters			R^2 (%)
	a	b	b/a	
		Glass surface		
8	0.103±0.0084	0.018±0.0108	0.18	98.6
11	0.084±0.0113	0.046±0.0144	0.55	97.0
13	0.098±0.0039	0.025±0.0050	0.26	99.7
15	0.092±0.0101	0.060±0.0129	0.65	98.2
18	0.120±0.0183	0.047±0.0236	0.39	95.5
		Steel surface		
8	0.064±0.0580	0.033±0.0074	0.52	98.6
11	0.048±0.0031	0.036±0.0040	0.75	99.5
13	0.050±0.0071	0.037±0.0092	0.74	97.2
15	0.041±0.0500	0.030±0.0064	0.75	98.0
18	0.075±0.0066	0.052±0.0850	0.69	98.9

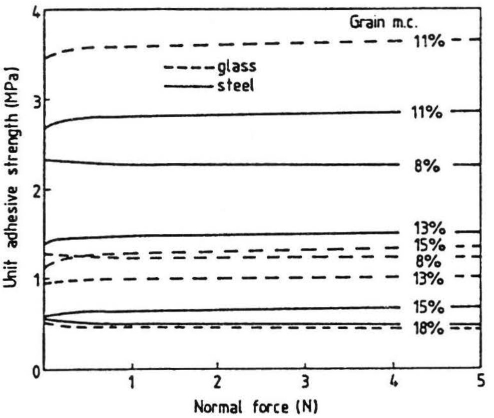

Fig. 3. Unit adhesive strength in relation to normal load for five levels of grain moisture content (cv. Grana wheat grain sliding on smooth glass and steel surfaces).

CONCLUSIONS

1. Coefficient of friction of a single grain increases rapidly with increasing roughness of sliding material. Minimum value of the friction coefficient (on the smooth plate) is determined by the unit adhesive strength. Maximum value of the friction coefficient is limited by shearing strength of seed coat.

2. For practical purposes, friction of a single wheat grain is adequately described by the Coulomb law but the experimental relationship between friction force and normal load is non-linear in the range of normal loadings below 0.2 N up to 0.5 N (depending on the moisture content of grain).

3. Application of the Kragelsky formula results in more accurate fitting of the equation to experimental data and allows for the physical interpretation of the process. The separation of friction force into adhesive and deformational components was found to be effective and reasonable in case of friction against the smooth surface. The unit adhesive strength was found independent of the normal load and showed the tendency to decrease with the increase of grain moisture content.

REFERENCES

1. **Horabik J., Molenda M.:** Force and contact area of wheat grain in friction. J. Agric. Eng. Res., 41, 33-42, 1988.
2. **Horabik J., Molenda M.:** Effects of moisture content on friction of wheat grain. Powder Handling and Processing, 1(3), 277-279, 1989.
3. **Bowden F.P.:** The Friction and Lubrication of Solids. Clanderon Press, Oxford, 1964.
4. **Kragelsky I.V., Dobytshin M.N., Kombalov V.S.:** Osnovy Rastsetov na Trienie i Iznos. Mashinostroenie, Moscow, 1977.
5. **Buckley D.H.:** Surface Effects in Adhesion, Friction, Wear and Lubrication. Elsevier Scientific Publishing Company, Amsterdam-Oxford-New York, 1981.